Computers and Chemical Engineering 35 (2011) 127–131

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Design of a modified sequential probability ratio test (SPRT) for pipeline leak detection

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ARTICLE INFO

Article history:
Received 5 March 2009
Received in revised form 17 May 2010
Accepted 17 June 2010
Available online 25 June 2010

Keywords:
Pipeline leak detection
CUSUM
SPRT

ABSTRACT

A classical SPRT likelihood test for sequential independently distributed data is often used in pipeline mass balance leak detection to distinguish between true leaks and false alarms in the minimum time consistent with a user defined error tolerance. However such time series data would not be expected to be independent, especially as it is often moving averaged to remove noise and unwanted transients. In this paper a modified SPRT test is derived using a simple Gaussian Markov process to model a correlated time series. Application of the modified test to correlated time series data is shown to reduce false alarms below that of a classical SPRT.

1. Introduction

There are a number of approaches to process fault detection such as standard statistical quality control (SQC) plots (Zhang, 1993), state estimators such as Kalman filters (KF) and likelihood tests. Of these the SPRT and Kalman filter approaches appear to be most promising (Alag, Agogino, & Morjaria, 2001; Zhang, 1993). Both the Kalman filter and SPRT can be shown to be optimal from a Bayesian standpoint, in that they both attempt to minimize objective functions that are aimed at reducing the probability of a false alarm while maximizing probability of detecting a genuine fault. The advantage of sequential methods such as the SPRT over batch processing is that all the samples are not required simultaneously to make a decision. The objective is to make such a decision in the minimum time consistent with the decision limits set for false alarm and missed detection rates.

The SPRT and KF approaches have some differences too. The KF is tuned beforehand and relies on prior estimates of the disturbance covariance matrices. It thus tends to be sensitive to these estimates and diverge if they are in error. Unlike KF, SPRT has the advantage of being robust, intuitive and easily tuned real time. It can also be shown to reach acceptance or rejection thresholds for fault detection with the minimum number of consecutive independent samples. However unlike KF it relies on the assumption these samples are iid (independently and identically distributed), an assumption which is often unjustified in practice.

In the case of mass balance pipeline leak detection the finite time constants of the system tend to produce a low pass filter that effectively correlates time series data (Montgomery, 2000). The effect of transients, linepacking and instrument noise are especially pronounced in the case of multiphase flow, making fault (leak) detection ineffective when standard statistical process control methods are employed. Thus such data is often processed using an SPRT which is further correlated by moving average filtering adopted in order to mitigate the effects of such transients (Turner, 1991).

In this document a modified SPRT test is developed that extends the classical SPRT to the case of (Gaussian Markov) random processes which are first order auto regressive (AR-1). The modified SPRT procedure reduces the number of erroneous alarms that would be created using a test that assumes an uncorrelated time series.

2. Review of SPRT

The SPRT distinguishes between two alternative one-sided hypotheses:

$$H_0 : \theta \le \theta_0, \quad H_1 : \theta \ge \theta_1, \quad \theta_0 < \theta_1 \tag{1}$$

The standard SPRT can be shown to minimize the number of samples required to distinguish between these two hypotheses, given pre assigned tolerance levels for making type I or type II errors. A type I error rejects H when its true, while a type II error accepts H when in fact it is false. The level of these probability thresholds depends on consequences of respective risks. For example, if the financial or human costs of a leak are extreme, then it

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doi:10.1016/j.compchemeng.2010.06.009

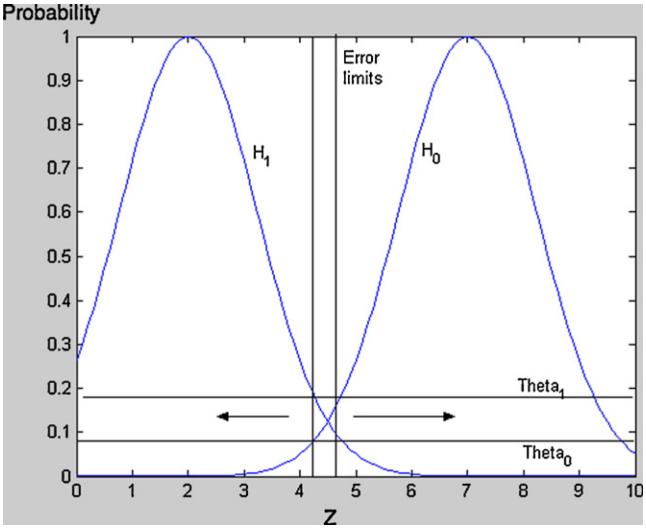

Fig. 1. Example of SPRT hypothesis test. To right of error limits H_0 is more likely, and on the left H_1 is. The limits on θ values set type I and type II errors.

may be better to tolerate the extra cost of coping with more false alarms. Fig. 1 illustrates the situation.

The plot is an example of how a classical SPRT test works. The x-axis represents some hypothetical values of Z derived from different hypotheses. θ_1 and θ_0 represent the thresholds on declaring an alarm condition. Moving left in the direction of the left arrow produces an increasing probability of the detection accepting H_1. θ_1 is the declaration limit on this association. The higher θ_1 the less likely a false alarm will be generated and the longer the resulting waiting time. The ratio of H_1 to H_0 gives the odds on this. Similarly as the θ_1 threshold is breached on the upside the Z statistic falls below the minimum value corresponding to θ_0 on the downside. This represents rejection of the H_0 hypothesis. Moving in the direction of the right arrow leads to acceptance of H_0. The range between θ_1 and θ_0 is the region where the test is operating but has not yet concluded. This corresponds to the error limits shown.

For a sequence of random variables $X_n = (x_1, x_2, x_3, \ldots x_n)$ the sequential application of the following likelihood ratio test minimizes the number of samples to make a decision subject to pre assigned type I and type II probabilities (Wald, 1947).

$$Z_n = \ln \frac{f(X_n|\theta_1, X_{n-1})}{f(X_n|\theta_0, X_{n-1})} \quad n \geq 1 \tag{2}$$

The hypothesis is accepted if $Z < b$ and rejected if $Z > a$. If neither is true the testing is continued. From Fig. 1 it is clear that a cascading test such as this repeatedly applied will narrow the distributions at each sample point, thus reducing the width of the ambiguous region between the error limits. For iid samples the criteria for preferring H_1 to H_0 can be written in incremental form as

$$Z_n = \sum_i^n \left\{ \ln\left[\frac{f(X_{i+1}|\theta_1, X_i)}{f(X_{i+1}|\theta_0, X_i)}\right] - \ln\left[\frac{f(X_i|\theta_1, X_{i-1})}{f(X_i|\theta_0, X_{i-1})}\right] \right\} \tag{3}$$

More simply this can be written as

$$Z_n = \sum_i \ln\left[\frac{f(X_{i+1}|\theta_1) f(X_i|\theta_0)}{f(X_{i+1}|\theta_0) f(X_i|\theta_1)}\right] \tag{4}$$

For iid distributions the distribution functions can be expressed as a product

$$f(X_i|\theta_1) = f(x_i|\theta_1) f(x_{i-1}|\theta_1) \ldots f(x_1|\theta_1) \tag{5}$$

Then Eq. (4) becomes

$$Z_n = \sum_i \ln\left[\frac{f(x_i|\theta_1, x_{i-1})}{f(x_i|\theta_0, x_{i-1})}\right] \tag{6}$$

The dependence on the past value is to indicate that as long as the time series has the Markov property the same formula clearly holds. Eq. (6) corresponds to a CUSUM plot which terminates if Z_n exceeds a threshold. The thresholds are related to the H_1 hypothesis type I (missed alarm) error probability p and type II (false alarm) error probabilities q by the Wald approximations (Wald, 1947).

$$\begin{aligned} b &\approx \ln\left(\frac{q}{1-p}\right) \\ a &\approx \ln\left(\frac{1-q}{p}\right) \end{aligned} \tag{7}$$

where b is the threshold for a missed alarm and a for a false alarm.

3. The leak detection problem

Assuming iid Gaussian random variables the probability density function of moving averaged leak data will be Gaussian. If the leakage at time, t, is $x(t)$ and the moving average is $m(t)$, then

$$f(x) = \frac{A}{\sigma_0(t)^2} e^{-\frac{[x(t)-m(t)]^2}{2\sigma_0(t)^2}} \tag{8}$$

where A is a constant and $\sigma_0(t)^2$ is local value of the leakage variance. Using Eq. (4) this results in the following expression for the CUSUM.

Disturbance of variance from σ_0 to σ_1:

$$Z_n = \sum_{i=1}^n \frac{1}{2}\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}(x_i - \mu_0)^2\right) - \ln\left[\frac{\sigma_1}{\sigma_0}\right] \tag{9}$$

Disturbance of mean from μ_0 to μ_1:

$$Z_n = \sum_{i=1}^n \frac{(\mu_1 - \mu_0)}{\sigma_0^2}\left[x_i - \mu_0 - \left(\frac{1}{2}\right)(\mu_1 - \mu_0)\right] \tag{10}$$

Unfortunately the test is less effective in this case. Leakage data samples, $x(t)$, are not iid between consecutive time intervals, since moving averaging correlates the data. To account for this case in the simplest manner, two further quantities are introduced. The one single lag autocorrelation of moving averaged data $R_{yy}(i+1, i)$ and the joint probability density function of consecutive samples, $f[y(i), y(i+1)]$.

For a process with spectral density $S_{xx}(\omega)$, moving averaged over the window – T to T the autocorrelation can be calculated using

$$R_{yy}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\omega)\left[\frac{\sin^2(T\omega)}{T^2\omega^2}\right] e^{j\omega\tau} d\omega \tag{11}$$

which is simply the convolution of the data with a moving average low pass filter. For data which is initially iid this reduces to (Papoulis, 1991)

$$R_{yy}(\tau) = \frac{1}{2T}\int_{-2T}^{2T}\left(1 - \frac{|\alpha|}{2T}\right) R_{xx}(\tau - \alpha) d\alpha \tag{12}$$

The correlation between consecutive samples at times t_1 and t_2 is simply $R = R_{yy}(t_2 - t_1)/\sigma^2$, or $R = R_{yy}(i+1, i)/\sigma^2$ for sampled data. The variance is simply $R_{yy}(0)$.

For Gaussian random variables (rv's) the joint pdf for subsequent data samples $y_1 = y(i)$ and $y_2 = y(i+1)$ is

$$f(y_1, y_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-R^2}} \exp\left[-\frac{1}{2(1-R^2)}\left(\frac{y_1^2}{\sigma_1^2} - 2\frac{Ry_1y_2}{\sigma_1\sigma_2} + \frac{y_2^2}{\sigma_2^2}\right)\right] \tag{13}$$