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Optimal profile of heat transfer pin fins under technological constraints

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ABSTRACT

Direct optimal control methods are used. The pin fin's volume is minimized for given heat flux value. Schmidt's criterion is not adopted. The optimal pin fin profile is neither parabolic nor circular. It consists of two regions. In the first region, close to the basis, the pin thickness decreases linearly. In the second region the pin thickness is constant or may decrease, depending on thermal loads and operation. The optimal control solution is usually singular but may be approximated by a bang–bang solution. The Schmidt criterion works better at larger heat flux values. Results obtained for specific assumptions adopted in the paper (fluid temperature 300 K, transverse Biot number ranging between 0.00041 and 0.041) are summarized next. For very small values of the heat flux (=0.1 W) the reduced minimum pin fin volume (i.e. the ratio between pin fin volume and the volume of a cylinder of similar length) is about one tenth. The technology and design constraints have important effects on the optimal pin fin's profile. The reduced minimum pin fin volume decreases from 0.30 to 0.20 when the maximum slope of the pin fin profile increases from 1 to 100. The reduced pin fin volume is a minimum minimorum for a maximum allowable pin fin temperature ranging between two and three times the fluid temperature.

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1. Introduction

Pin fins or spines are common devices used to enhance the heat transfer. Information about their common utilizations and how they are usually optimized is provided in section S1 of the ESM (Electronic Supplementary Material). Different simplifying hypotheses have been adopted in previous pin fin optimization works. They include the 1D approximation, the Schmidt criterion [1], the "length-of-arc assumption" [2], usage of constant properties (including the constant heat transfer coefficient) along the pin fin length [3], and the assumption of zero heat flux at pin fin's tip [2]. Taking into account the large number of possible combinations among these simplifying assumptions and the large number of objective functions and constraints, it is obvious that the number of optimum design solutions reported in literature is large. In some particular cases they seem to contradict each other. For instance, the classical, optimum parabolic pin profile is found by using the Schmidt's criterion and the "length-of-arc assumption" [1]. However, when the "length-of-arc assumption" is relaxed, the optimum profile is a circular arc [4]. Careful examination on the basic assumptions should be made before results are interpreted.

This paper proposes a procedure for the optimization of pin fin's profile. The objective is to minimize the material volume. Details about the assumptions adopted here are given in Section 2. The paper brings four novelties. First, pin fins of circular cross-section are usually considered [5,6]. Here elliptical pin fins are treated. Second, indirect methods based on the Pontryagin's Maximum Principle used in Ref. [7] have the advantage of the elegant theoretical formulation. The main difficulties arise in case of problems with many constraints, where switching between singular arcs is necessary. Here we are using a direct method based on non-linear programming which makes easier solving constrained optimization problems. Third, the pin fin diameter has been used as a control in previous optimal control approaches (see e.g. Ref. [2]). This does not allow including the arc length in calculation since it involves the space derivative of the control, a case not covered by standard optimal control theory. Here we use the profile slope as a control. This allows including the pin fin's diameter (or other cross section's characteristic length) among the state variables. Fourth, the usual constraint considered in previous studies refers to pin fin length [2]. This constraint is used here, too. In addition, technological

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Nomenclature

Latin letters

A	surface area (m^2)
a	length of ellipse's semi-major axis (m)
b	length of ellipse's semi-minor axis (m)
f	function defined by Eq. (6) (m^3)
h	convection heat transfer coefficient ($W/(m^2K)$)
P	perimeter length (m)
Q	heat flux (W)
L	pin fin length (m)
T	pin fin temperature (K)
T_∞	fluid temperature (K)
u	dimensionless function entering Eq. (4)
$\widetilde{u}$	dimensionless control defined by Eq. (10c)
V	pin fin volume (m^3)
$\widetilde{V}$	dimensionless pin fin volume defined by Eq. (9d)
v	dimensionless function entering Eq. (4)
$\widetilde{v}$	dimensionless control defined by Eq. (10d)
z	spatial coordinate (m)

Greek letters

λ	thermal conductivity of pin fin's material (W/(mK))
θ	dimensionless pin fin temperature
θ_∞	dimensionless fluid temperature
$\widetilde{\theta}$	dimensionless variable defined by Eq. (10a)
ξ	dimensionless space variable

Subscripts

0	pin fin basis
L	tip of pin fin
max	maximum
min	minimum
opt	optimum
ref	reference

constraints are taken into account, including design as well as operational constraints, such as the specified maximum slope of pin fin's profile and the maximum allowable temperature. They are rather easily implemented within direct optimal control methods.

2. Methodology

2.1. Geometry

Elliptical pin fins of length L are considered in this paper. A more general approach of pin fin geometry is described in section S2 of ESM.

2.2. Heat transfer model

Several usual hypotheses are adopted. The pin fin's material is homogeneous and isotropic. The 1D approximation is adopted and z denotes the coordinate along the pin fin, with $z = 0$ associated with pin fin's basis. The pin fin is surrounded by a fluid at constant and uniform temperature T_∞. The pin fin temperature at basis level is $T(z=0) = T_0(>T_\infty)$. Thus, conduction heat transfer takes place inside the pin fin in the direction of increasing z. Convection heat transfer occurs from pin fin's surface towards the fluid. The processes of heat conduction along the pin and heat convection from the pin to the fluid are steady-state. Constant values are considered here for the convection heat transfer coefficient h along the pin length. Comments and motivation concerning these assumptions are presented in section S3 of ESM where a literature survey is also included.

The steady-state energy balance for a slide of thickness dz around the cross section through the pin fin at coordinate z gives [8]:

$$\frac{d}{dz}\left[\lambda A(z)\frac{dT}{dz}\right] - hP(z)(T - T_\infty) = 0 \tag{1}$$

where λ is the thermal conductivity of pin's material, assumed constant, while $A(z)$ and $P(z)$ denote the cross section's surface area and perimeter, respectively. The first term in the l.h.s. member of Eq. (1) describes the heat conduction within the pin while the second term describes the convection heat transfer from the pin towards the fluid.

The heat flux transferred from the pin towards the fluid is given by an integral over the pin's surface. Since the lateral surface of a slide of thickness dz is well approximated by $P(z)dz$, then the heat flux Q is given by Ref. [8]:

$$Q = \int_0^L hP(z)(T - T_\infty)dz \tag{2}$$

Since no heat source exists inside the pin, the heat flux Q equals the heat flux Q_0 transferred by conduction through pin's basis:

$$Q_0 = -\lambda A(z=0)\left.\frac{dT}{dz}\right|_{z=0} \quad (=Q) \tag{3}$$

2.3. Optimal control problem

Two objective functions are considered. They are inspired by practical considerations taking into account operational requirements and financial investments.

In case of high technology applications characterized by severe operational conditions the investments aspects may become secondary. Thus, the first objective function is the transferred heat flux, which is maximized. The optimal control solution is rather trivial and is shortly described in section S4 of ESM.

Routine large-scale applications, with rather common operational conditions, require an economic analysis, which involves investments costs (related to the amount of material used and the pin fin manufacturing process) and financial savings during operation (related to the economical effect which heating or cooling a fluid may have). Information about the economics of pin fin's manufacturing process and operation is not available here. However, the cost of the material is proportional in first approximation with its volume. Therefore, material cost minimization is associated with minimization of pin fin volume. The second objective function is the pin fin's material volume V which is minimized (for given value of the transferred heat flux Q_0). The volume V is given by:

$$V = \int_0^L A(z)dz = A_0\int_0^L u(z)v(z)dz \tag{4}$$

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