



# Linear algebra solution to psychrometric analysis of air-conditioning systems



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## ABSTRACT

The typical air conditioning steady-state processes are graphically represented by straight or curve lines on the psychrometric chart. Neglecting the sensible heat of the moisture results in decoupling the sensible and the latent heat, that results in linear variation of the enthalpy on the psychrometric chart. The vapor saturation curve may also be linearized by using Newton's method. If the mass flow rates of the dry air are known and if the computational causality is assigned to correspond to the physical causality (i.e. if a direct modeling problem is treated), then the steady-state models of the psychrometric processes become linear algebraic equations in the vector space defined by the dry bulb temperature and the humidity ratio. Coupling these models to describe a complex HVAC (heating ventilation air-conditioning) system results in a system of linear equations that solves a direct (or psychrometric analysis) problem in which the inputs of the model are a subset of the set of independent variables of the psychical process, the outputs of the model are a subset of the set of the dependent variables of the physical process, and the unknowns are the psychrometric states of the moist air. The algorithm that implements this method represents a computational alternative to graphical representations and manual solutions to psychrometric analysis of air-conditioning systems.

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## 1. Introduction

### 1.1. Causality and types of modeling problems

The models of psychrometric processes encountered in HVAC (heating, ventilation and air-conditioning) systems are based on mass and energy balance equations. These equations do not reveal the computational causality, i.e. the input–output relationship. A physical system is characterized by its causality: the values of the dependent variables (outputs) follow in time the variation of the independent variables (inputs). A mathematical model is also a system that connects, through computations, the inputs and the outputs: the inputs are provided to a computational algorithm (implemented in computer languages by functions, subroutines, blocks, objects, etc.) in order to obtain the outputs [1]. Depending on the relation between the physical causality and the computational causality, the modeling problems may be classified in (Table 1):

- Direct: when the physical and the computational causalities are the same, i.e. the inputs and the outputs of the model are a

subset of the inputs and of the outputs of the system, respectively.

- Inverse: when the physical and the computational causalities are different. There are two types of inverse problems:
  - parameter identification: given the inputs, the outputs, and the structure of the model (derived from physical considerations or achieved heuristically), obtain the parameters of the model;
  - process control: knowing the desired output (i.e. the set-point) and the model of the process, find the input (i.e. the command).

When the structure of the model and its parameters are known, the model is a white box. When the parameters are identified for a structure of the model that has physical significance, the model is a gray box; if the structure of the model is obtained heuristically, the model is a black box.

### 1.2. Dynamic models for energy estimation vs. steady-state models for psychrometric analysis

If the models take into account the time variation of the accumulated mass and energy, then the model is dynamic. If this time variation is not taken into account, then the steady-state is

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Nomenclature	
$A$	area [m <sup>2</sup> ]
$c$	specific heat capacity [kJ/kg] – of dry air, if no subscript
$E$	heat exchanger effectiveness [–]
$h$	specific enthalpy [kJ/kg]
$H$	heating
$l$	latent heat for vaporization [kJ/kg]
$m$	mass [kg]
$M$	molar mass [kg/kmol]
$n$	molar fraction [–]
NTU	number of transfer units [–]
$p$	pressure [Pa]
$R$	ideal gas constant [kJ/kmol K]
$T$	temperature [K]
$U$	overall heat-transfer coefficient [kW/m <sup>2</sup> K]
$w$	humidity ratio [kg <sub>w</sub> /kg <sub>da</sub> ]
<i>Dotted variables</i>	
$\dot{H}$	enthalpy rate [kW]
$\dot{m}$	mass flow rate [kg/s] – of dry air, if no subscript
$\dot{Q}$	heat flow rate [kW]
<i>Bold letters</i>	
$\mathbf{A}$	square matrix of the coefficients of the model
$\mathbf{A}_{mn}$	element ( $m,n$ ) of the matrix of the coefficients of the model
$\mathbf{b}$	vector of inputs
$\mathbf{b}_m$	element $m$ of the vector of inputs
$\mathbf{x}$	vector of unknowns
<i>Greek letters</i>	
$B$	by-pass factor [–]
$\Delta$	difference
$\epsilon$	mixing coefficient, contact factor [–]
$\theta$	temperature [°C]
$\varphi$	relative humidity [%]
<i>Subscripts</i>	
aux	auxiliary (solar, occupants, electrical, etc.)
da	dry air
inf	air infiltration
$l$	latent
$o$	outdoor
$s$	saturation (temperature, humidity ratio)Sensible (heat, enthalpy)
$w$	water vapor
$x$	exchanger
<i>Superscripts</i>	
$T$	transpose of a vector or a matrix
'	derivative of a function
$0$	initial value in an iteration

modeled. Generally, the direct problems are well posed for both dynamic and steady-state models while the inverse problems are ill posed [2,3]. Dynamic simulation of HVAC systems coupled with the building is a subject widely studied and it is implemented in many software packages [4] (among them TRNSYS [5,33], EnergyPlus [6], ESP-r [7]). Co-simulation, in which different simulation tools running simultaneously and exchanging information simulate different components, benefits from the capabilities of specialized software environments [8,9]. This approach is used for the estimation of the energy consumption and of the thermal loads [10,2].

In steady-state, the inverse modeling problems are well posed, which explains their success in design [11,12]. Important HVAC engineering problems may be seen as combinations of direct and inverse modeling problems represented on psychrometric charts:

analysis and solution verification [13–15], parameter optimization in design [16–19], performance evaluation [20–24], efficient control [25], fault detection and diagnosis [26], etc.

### 1.3. Psychrometric analysis of HVAC systems

Since 1904, when W. H. Carrier introduced it, the most used tool for analyzing the HVAC systems coupled to buildings is the graphical representation of steady-state processes of humid air on the psychrometric chart [27]. The typical processes of moist air transformations are presented in almost any primer on air-conditioning by using the sensible and latent heat balance in steady-state [28,34]. Currently, most practitioners rely on psychrometric software (like ASHRAE Psychrometric Chart App, Trane

**Table 1**  
Types of modeling problem.

Physical system		Type of modeling problem		
		Direct	Inverse	
$f(\cdot)$	Model structure	$y \leftarrow f(x a)$	$a \leftarrow \{y, f(x \cdot)\}$	$x \leftarrow \{y, f(\cdot a)\}$
$a$	Parameters of the model	Simulation	Identification	Control
$x$	Measured inputs of the physical system	White box	Black box	Gray box
$y$	Measured outputs of the physical system	Physics	Heuristic	Physics
		Known	Output	Output
		Input	Input	Input
		Output	Input	Input
			Input	Input
				Feed-back
				Known
				Output
				Input

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