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Two energy conservation principles in convective heat transfer optimization

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ABSTRACT

Improving heat transfer performance is very beneficial to energy conservation because heat transfer processes widely existed in energy utilization systems. In this contribution, in order to effectively optimize convective heat transfer, such two principles as the field synergy principle and the entransy dissipation extremum principle are investigated to reveal the physical nature of the entransy dissipation and its intrinsic relationship with the field synergy degree. We first established the variational relations of the entransy dissipation and the field synergy degree with the heat transfer performance, and then derived the optimization equation of the field synergy principle and made comparison with that of the entransy dissipation extremum principle. Finally the theoretical analysis is then validated by the optimization results in both a fin-and-flat tube heat exchanger and a foursquare cavity. The results show that, for prescribed temperature boundary conditions, the above two optimization principles both aim at maximizing the total heat flow rate and their optimization equations can effectively obtain the best flow pattern. However, for given heat flux boundary conditions, only the optimization equation based on the entransy dissipation extremum principle intends to minimize the heat transfer temperature difference and could get the optimal velocity and temperature fields.

1. Introduction

As the worldwide energy crisis is becoming serious day by day, researchers and engineers have been paying more and more attention to both energy conservation and equipment size reduction. Because nearly 80% of the total energy consumption is related to heat transfer, enhancing and especially optimizing heat transfer are highly desired for effective energy utilization. In the past several decades, many heat transfer enhancement technologies have been brought forward to raise the efficiency of heat transfer, such as extended surfaces, inserts, swirlers and external electrical/magnetic forces [1–4]. The physical mechanisms of these technologies can be classified into such three categories as reducing thermal boundary layer, mixing the main and near-wall flows and creating secondary flow to raise the turbulence intensity of flow [1,2]. However, as Bergles and Webb [3–5] said, these mechanisms were either empirical or semi-empirical, which were mainly obtained by analyzing and summarizing experimental or numerical results, and hence we still lack some universal theories derived from the nature of heat transfer. Moreover, accompanying heat transfer enhancement, the flow resistance, i.e. the pumping power, is often increased largely by using the aforementioned methods, which is adverse to energy conservation and consequently restricts the wide applications of those heat transfer enhancement technologies. Therefore, heat transfer optimization should be taken into consideration, that is, the heat transfer should be enhanced on the premise of some constrains, e.g. a fixed pumping power, and it is definitely favorable to energy conservation.

From the energy conservation equation in convective heat transfer, we can readily find that heat transfer performance is determined by such physical parameters as viscosity, thermal conductivity, velocity and temperature. Based on the variational principle, we will obtain the variational form with the heat transfer optimization aim of minimizing the temperature difference at a given heat transfer rate, which is expressed as,

$$\delta(\Delta T) = \delta\left(x, y, z, T, U, \lambda, \mu, \rho, c_p, \cdots\right) \tag{1}$$

or maximizing the heat transfer rate at a given temperature difference.

$$\delta(Q) = \delta\left(x, y, z, T, U, \lambda, \mu, \rho, c_p, \cdots\right) \tag{2}$$

However, since neither the temperature difference ΔT nor the heat flow rate Q is a local parameter, it is difficult to establish the theoretical relationship of either the temperature difference or the boundary heat transfer rate with other related local physical

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Nomenclature

A, B, C, A', B', C'	Lagrange multiplicator
F	additional volume force per unit volume, N m^{-3}
c_p	specific heat capacity, J kg^{-1} K^{-1}
ρ	density, kg m^{-3}
μ	dynamic viscosity, kg m^{-1} s^{-1}
λ	thermal conductivity, W m^{-1} K^{-1}
U	velocity vector, m s^{-1}
$\overline{U}$	dimensionless velocity
u, v, w	velocity component in x, y and z directions, m s^{-1}
P	pressure, Pa
T	temperature, K
$\overline{T}$	dimensionless temperature
S	heat transfer area, m^2
V	volume, m^3
$\overline{V}$	dimensionless volume
V_0	unit volume, m^3
β	field synergy angle between U and ∇T, °
n	outward normal unit vector
q''	heat flux, W m^{-2}
q_v	heat generated per unit volume, W m^{-3}
Q	total heat flow rate, W
x, y, z	Cartesian coordinates, m
D	characteristic length, m
L_f	fin length, m
F_p	fin pitch, m
δ_f	fin thickness, m
δ_w	wall thickness, m
W_t	tube width, m
T_p	tube pitch, m
Re	Reynolds number
Pr	Prandtl number
Nu	Nusselt number
ϕ_h	entransy dissipation rate per unit volume, W K m^{-3}
ϕ_m	viscous dissipation rate per unit volume, W m^{-3}
Φ_m	viscous dissipation rate, W
Π	Lagrange function
∇	divergence operator
Subscript	
b	boundary
m	average
in	inlet
out	outlet
f	flow

parameters, so the variational methods shown in Eqs. (1) and (2) are not practically useable. On the other hand, if introducing some other parameters composed of local physical variables, to represent the heat transfer performance, we may obtain the optimization equations for convective heat transfer based on the variational method [6,7].

After attentively reexamining the influence of both the velocity and the temperature fields on the heat transfer performance in a boundary-layer flow, Guo et al. [8,9] developed the field synergy principle (FSP) to analyze and evaluate the convective heat transfer performance. It states that the convective heat transfer performance depends on not only the magnitudes of both the velocity vector and the temperature gradient, but also their synergy, i.e. the intersection angle between them. In addition, the field synergy number, a dimensionless parameter, was introduced to quantitatively evaluate the field synergy between the velocity and the temperature gradient fields. From then on, the FSP has been widely used in both academic and engineering fields [6,15–20]. For instance, Zhao et al. [10] revealed that heat transfer can be significantly enhanced when the flow direction is parallel to the applied temperature gradient, Oppositely, if a flow is normal to a temperature gradient, the flow will not make any contribution to the heat transfer occurring in the temperature gradient direction. This conclusion was also experimentally verified by Ma et al. [11]. Furthermore, after making a large amount of numerical and experimental verifications, Tao et al. [12–14] extended the FSP from the parabolic flow to the elliptic flow, and found that the FSP could unify the aforementioned three physical mechanisms of heat transfer enhancement, i.e. reducing thermal boundary layer, mixing the main and near-wall flows, and creating secondary flow. However, so far when using the FSP, researchers and engineers generally have to develop some structures first, then numerically or experimentally obtain the temperature gradient and the velocity field, and finally use the FSP to assess whether the field synergy degree becomes better or worse. That means it is actually a *try and error* method, which hardly obtain the optimal flow pattern with the best heat transfer performance.

In order to overcome this problem, by analogy with the phenomena of electricity conduction, Guo et al. [21] introduced a new physical quantity, termed *entransy*, to describe the heat transfer ability of an object or a system, deduced the expression of entransy dissipation during heat transfer to quantitatively evaluate the irreversibility of heat transfer, and finally developed the entransy dissipation extremum principle (EDEP) for the optimization of heat transfer. The entransy theory and the corresponding optimization method have been validated and applied to all the three modes of heat transfer, i.e. heat conduction [22], convective heat transfer [7,23] and thermal radiation [24,25]. Moreover, the optimized results obtained by the EDEP were also compared to those optimized by the minimum entropy generation principle [7], another widely used heat transfer optimization method [26,27] proposed by Bejan [28,29].

From the above discussion, it is concluded that both the FSP and the EDEP have the goal of optimizing heat transfer. However, the connection of the extremum of entransy dissipation with the convective heat transfer performance has not been constructed until now, and the intrinsic relations of the EDEP and the FSP with the heat transfer performance have not yet been discussed in any literature. Therefore, this contribution first theoretically analyzes the relationships of the FSP and the EDEP with the convective heat transfer performance, takes the field synergy degree as an optimization criterion to derive an Euler's equation by the variational method, then uses both this equation and the one deduced based on the EDEP, respectively, to optimize the convective heat transfer processes in both a fin-and-flat tube heat exchanger and a four-square cavity under different boundary conditions, and at last illustrates the relationship and the difference between these two optimization methods based on the theoretical and numerical results.

2. Optimization principles for convective heat transfer

2.1. Relation of heat transfer performance to field synergy degree

For a steady-state incompressible convective heat transfer process without any internal heat source in the entire domain, after ignoring the viscous dissipation, the energy conservation equation is expressed as