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Review

Review of mathematical models of future oil supply: Historical overview and synthesizing critique

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ABSTRACT

This review paper assesses oil supply modeling techniques and critically evaluates their usefulness in projecting future oil production. It reviews models that project future rates of oil production, but does not address estimation of oil resources. The following types of models are reviewed: the Hubbert method; other curve-fitting methods such as exponential and Gaussian models; simulation models of resource discovery and extraction; and data-rich "bottom-up" models. Economic models are reviewed more briefly. Forty-five mathematical models of oil depletion of the last century are classified along four dimensions of variability: emphasis on physical or economic aspects of oil production; model scale; hypothetical or mechanistic orientation; and complexity. Models based on quite disparate assumptions (e.g., physical simulation vs. economic optimal depletion) have produced approximately bell-shaped production profiles, but data do not support assertions that any one model type is most useful for forecasting future oil production. In fact, evidence suggests that existing models have fared poorly in predicting global oil production. The greatest promise for future developments in oil depletion modeling lies in simulation models that combine both physical and economic aspects of oil production.

1. Introduction

Concern about the availability of oil emerged soon after the birth of the oil industry and has resurfaced repeatedly in predictions of exhaustion of oil resources. While these projections have generally proven incorrect (sometimes spectacularly so), the future of oil remains uncertain. This is because, as Adelman [1] argues, the oil industry is fundamentally "a tug-of-war between depletion and knowledge." Although knowledge has won out over depletion for the last 150 years, allowing us to continually increase oil production, there is uncertainty about how much longer this will continue.

Those who attempt to model oil depletion face two questions. First, *how much recoverable oil exists?* This question requires estimating ultimately recoverable resources (URR), or the amount of oil that can be economically produced over all time. Second, *what path will production take over time?* This question requires converting an estimate of URR into an estimate of future rates of oil production. This review discusses mathematical methods of addressing this second question.

Quantitative understanding of oil depletion has increased significantly over the last century. Calculations of the exhaustion time of oil reserves were performed as early as 1909 [2]. By mid-century, methods of predicting field-level production were used in evaluating producing fields [3], and statistical methods were developed to better project how much oil is likely to be found in a given region [4]. In the 1950s and 1960s, curve-fitting techniques were used to forecast petroleum production [5]. After the oil crisis of 1973, the problem of oil depletion received significant attention from economists, elevating resource depletion to a topic of vigorous theoretical exploration [6]. And finally, the 1970s and 1980s saw increasing focus on econometric modeling of oil discovery and extraction [7]. Academic interest in oil depletion waned after the oil price decline of the mid 1980s, resulting in a decline in academic interest until the early 2000s.

This paper has two goals. First, it provides a systematic review of oil depletion models produced to date. This serves to make obscure past works (often difficult to find) available to a wider audience so as to limit repetition of past efforts. Second, this paper provides synthesizing critique of previous modeling efforts, with the aim of improving future oil depletion modeling.

To limit the review scope to a tractable size, this paper does not review models used for predicting production from individual fields (i.e. exponential or hyperbolic decline curves, e.g., [3]). It also does not review statistical "discovery process" models, because they are more typically used to estimate URR than to project future production paths [8]. And our focus in econometric modeling

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Nomenclature

t	time
t_0	initial time period
t_{peak}	year of peak oil production
t_{ex}	year of exhaustion of oil resources
$P(t)$ or P	oil production in a given year t, equal to $Q'(t)$ or dQ/dt
P_0	oil production in initial year t_0
P_{peak}	oil production in year t_{peak}, or maximum oil production rate
$Q(t)$ or Q	cumulative oil production to year t, equal to sum of $P(t)$ from years t_0 to t
Q_∞	ultimately recoverable resources (URR), equal to sum of $P(t)$ from years t_0 to ∞.
$D(t)$	cumulative oil discoveries to year t
$R(t)$ or R	oil reserves in year t
$M(t)$ or M	remaining oil resources in year t, equal to $Q_\infty - Q(t)$. $M(t)$ is larger than $R(t)$ due to undiscovered oil and reserve growth.
r_{inc}	rate of increase of oil production
r_{dec}	rate of decrease of oil production

largely focuses on "hybrid" models that contain geological or other non-economic factors in addition to economic ones.

The mathematical notation used in models has been altered, where possible, to be consistent across all studies (e.g., URR is represented by Q_∞ in all models).

2. Simple models of oil depletion: reserve-to-production and curve-fitting models

The simplest models of future oil production are reserve-to-production (R/P) models. The number of years until reserve exhaustion (t_{ex}) is calculated by dividing an estimate of current reserves (R), or sometimes remaining resources (M), by current production (P):

$$t_{ex} = \frac{R}{P}, \tag{1}$$

or,

$$t_{ex} = \frac{M}{P}. \tag{2}$$

Because M accounts for reserve growth and yet-to-find oil, the estimate of t_{ex} from Eq. (2) will be larger.

Variations of the R/P methodology that include production growth have been used since 1909 when Day [2,9] published R/P calculations that accounted for demand, causing significant concern [10]. If production grows exponentially at rate r after the initial model year t_0 then

$$\int_{t_0}^{t_{ex}} Pe^{rt} dt = R, \tag{3}$$

or if one solves for t_{ex},[1]

$$t_{ex} = \frac{1}{r}\ln\left(\frac{Rr}{P}+1\right). \tag{4}$$

[1] Of course, M could also be used in place of R in this model as well.

Reserve-to-production methods have been used continuously since, especially in popular accounts of oil depletion (most frequently in support of optimistic assessments of resource availability).[2]

R/P-derived predictions bear little relationship to observed production profiles. This is because reserve-to-production modeling is "a fallacious approach based on circular reasoning": reserves are estimates of what is currently thought to be economically producible at a given level of confidence, not the total oil in place [12]. Thus, R/P measures the *inventory* of delineated petroleum deposits, not the oil resource.

Curve-fitting models of oil production have been used since the 1950s. A variety of models exist, but their general approach is as follows.

1. Define a mathematical function to statistically fit to historical production data.
2. Include constraints to improve the quality of model fit.
3. Fit the constrained model to historical data to project future production.

Curve-fitting models vary in the function used, in the use of URR as a constraint and in the usage (or not) of symmetric model functions.

2.1. Hubbert's logistic model

M. King Hubbert produced a well-known projection of future US oil production in 1956 [5], utilizing a bell-shaped curve to predict future production. Some have argued that Hubbert's projections were unprecedented, but others made similar projects in the same time period. In 1953, Ayres [13] predicted that United States peak production of oil would occur in 1960 or 1970 depending on the level of ultimate recovery (100 or 200 Gbbl URR, respectively). Also, in 1952 the study *Resources for Freedom* predicted peaks of 1963 to 1967 in two scenarios [14].

In March of 1956, Hubbert predicted that US oil production would peak between 1965 and 1970 [5]. These two projections differed by the value of URR used to constrain the production curve (150 and 200 Gbbl, respectively). This prediction gained notoriety when United States production peaked in 1970. His first predictions were based on hand-drawn, slightly asymmetric production projections.

In 1959, Hubbert outlined his mathematical method: he fit the logistic function (a "sigmoid" curve) to cumulative oil discoveries. He extrapolated this curve to find the asymptote of cumulative discoveries (URR), which he then used to constrain the corresponding production curve. In 1980, he published a full derivation of his logistic model [15].

Hubbert's mathematical model assumes the following.

1. Yearly production is modeled as the first derivative of the logistic function.
2. The production profile is symmetric (i.e., maximum production occurs when the resource is half depleted and its functional form is equivalent on both sides of the curve).
3. Production follows discovery with a constant time lag.
4. Production increases and decreases in a single cycle without multiple peaks.

While he was often critiqued for the validity of these assumptions, Hubbert noted frequently that these were only simplifying

[2] Journalistic use of R/P occurred as early as 1920, when the New York Times [11] reported a Bureau of Mines calculation that the United States "has only an 18-year supply."