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A time/space separation-based Hammerstein modeling approach for nonlinear distributed parameter processes

Chenkun Qi, Han-Xiong Li *

Department of Manufacturing Eng & Eng Management, City University of Hong Kong, Kowloon, Hong Kong SAR, PR China

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ABSTRACT

Modeling of distributed parameter systems (DPSs) is very difficult because of their infinite-dimensional, spatio-temporal nature and nonlinearities. A low-order, simple nonlinear and parsimonious model is often required in real applications. In this study, a time/space separation-based Hammerstein modeling approach is proposed for unknown nonlinear DPS. Firstly, the Karhunen-Loève (KL) method is used for the time/space separation, where the spatio-temporal output is decomposed into a few dominant spatial basis functions with temporal coefficients. Secondly, a low-order parsimonious Hammerstein model is identified from the low-dimensional data to reconstruct the system dynamics, where the parsimonious model structure is determined by the orthogonal forward regression and the parameters are estimated using the least squares estimation and the singular value decomposition. The algorithm does not require nonlinear optimization and it is numerically robust. This modeling is very suitable for control design. The simulations are presented to show the effectiveness of the proposed modeling method.

1. Introduction

Many technological needs such as semiconductor manufacturing, nanotechnology and biotechnology have motivated control of material microstructure, spatial profiles and product size distributions (Christofides, 2001a). These physical, chemical or biological processes all lead to so-called distributed parameter systems (DPSs) where the input and output vary both temporally and spatially. Recent developments in sensor, actuator and computing technology make the modeling and control of spatio-temporal systems feasible and practical. Obtaining their spatio-temporal models is the first step for the model-based control. System identification has been extensively studied for lumped parameter systems. However, only a few studies have been reported for distributed parameter systems. The main difficulty comes from the time/space coupling of distributed parameter systems. In this study, we will focus on the spatio-temporal modeling problem for unknown nonlinear distributed parameter systems.

The first-principle modeling typically leads to various partial differential equation (PDE). However their infinite-dimensionality does not allow their direct use for the design of implemental controllers. Currently, there are two DPS control methodologies: "reduce-then-design" and "design-then-reduce". Most of DPS control methods first reduce the model dimension, and then perform the control design by extending the finite-dimensional control methods. In practice, a low-dimensional, simple nonlinear model with parsimonious structure is often needed.

- Conventional time/space discretization approaches such as finite difference for model reduction, can easily transform the PDE to the ordinary differential equation (ODE). However, they often lead to a high-order model, which is not very suitable for synthesizing implemental controller if the model is not further reduced.
- Advanced model reduction approaches are based on time/space separation using spatial basis function expansion. The idea comes from Fourier series expansion. A spatio-temporal variable can be expanded onto an infinite number of spatial basis functions with temporal coefficients. After choosing the proper finite number of spatial basis functions, a finite-dimensional temporal coefficients model can be obtained by minimizing the approximation error (i.e., residual). This is so-called the weighted residual method (WRM) (Ray, 1981). The modeling accuracy and efficiency is very dependent on the choice of basis functions. Unfortunately local basis function expansion in finite element method will also lead to a high-order ODE model. However global basis function expansion in spectral method (Boyd, 2001) can derive an accurate low-order ODE model, which is suitable for real-time control. Some global basis functions include Fourier series (Boyd, 2001), orthogonal polynomials (Sadek & Bokhari, 1998), and eigenfunctions of the system (Christofides, 2001b; Ray, 1981). However these general basis functions are not optimal in the sense that the model dimension is not the lowest given the desired model accu-

* Corresponding author. Tel.: +852 27888435; fax: +852 27888423.
E-mail address: mehxli@cityu.edu.hk (H.-X. Li).

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racy. Karhunen-Loève (KL) decomposition (Baker & Christofides, 2000; Park & Cho, 1996; Sirovich, 1991) is a popular approach to find the principal spatial structures from the data. Among all linear basis function expansion, KL basis functions are optimal which can often give a lower dimensional model. In addition, the model order can be further reduced when using nonlinear WRM such as the inertial manifold method (Christofides & Daoutidis, 1997).

However, all these model reduction methods require that the PDE description of the system is accurately known. In many cases, it is difficult to obtain an exact PDE description of the industrial process and the data based identification has to be employed. If the PDE structure of the system is known and only some parameters are unknown, then these parameters can be estimated from the process data (e.g., Banks & Kunisch, 1989; Demetriou & Rosen, 2001). If some nonlinear terms of the PDE are also unknown, then these nonlinearities and parameters can be approximately identified using a hybrid neural and spectral method (Deng, Li & Chen, 2005). For completely unknown processes which widely exist in the industry, the black-box identification has to be used. Recently, some black-box modeling approaches have been proposed.

- The DPS identification methods using linear Green's function model have been studied by Gay and Ray (1995), Zheng, Hoo, and Piovoso (2002) and Zheng and Hoo (2002, 2004). However, a linear model is only able to approximate the nonlinear system around a given working point.
- The time/space discretization of a DPS using finite difference will lead to a so-called lattice dynamical system. The identification of unknown lattice dynamical system has been studied in Guo & Billings (2007) and Parlitz & Merkwirth (2000). However the model may be of a high-dimension.
- The time/space separation of a DPS using spatial basis function expansion will lead to some ordinary differential equations. After choosing the proper basis functions, the corresponding unknown ODE can be estimated using traditional system identification techniques. Some identification methods such as nonlinear autoregressive with exogenous input (NARX) model with local finite element basis functions (Coca & Billings, 2002), and neural networks with global Karhunen-Loève basis functions (Zhou, Liu, Dai & Yuan, 1996) have been studied. However such models may be very complex which are not easily used for practical control.

Hammerstein models are widely used in engineering practice due to their capability of approximating many nonlinear industrial processes and simple block-oriented nonlinear structure (i.e., a nonlinear static element in series with a linear dynamic system). Examples are the modeling of the pH neutralization process, the continuous stirred tank reactor and distillation columns. Because a linear structure model can be derived from the block-oriented nonlinear structure, the linear control design can be easily extended to Hammerstein models. Successful control applications have been reported for traditional ODE processes (Fruzzetti, Palazoglu & McDonald, 1997; Samuelsson, Norlander & Carlsson, 2005). However, because the Hammerstein model does not have inherent capability to process spatio-temporal information, few studies can be found in the DPS. Recently the spatio-temporal Hammerstein model is proposed for nonlinear DPS by adding the spatial variable into the traditional Hammerstein model (Qi, Zhang & Li, 2009), which is constructed with a static nonlinearity followed by a Green's function as a dynamic linear DPS.

Many approaches to the identification of Hammerstein models can be found in the literature (e.g., Bai, 1998; Chen, 2004; Ding & Chen, 2005; Greblicki, 2006; Gómez & Baeyens, 2004; Narendra & Gallman, 1966; Stoica & Söderström, 1982; Vörös, 2003; Zhu, 2000). It is notable that an algorithm based on the least squares estimation and the singular value decomposition (LSE–SVD) is proposed for Hammerstein-Wiener systems (Bai, 1998) and extendedly studied for Hammerstein systems (Gómez & Baeyens, 2004).

This algorithm can avoid the local minima since it does not require any nonlinear optimization. However, in Bai (1998) the model structure is assumed to be known in advance and only unknown parameters need to be estimated. In many cases, the structure is often unknown and the model terms and orders have to be determined carefully. If the structure is inappropriate, it is very difficult to guarantee the modeling performance. In Gómez and Baeyens (2004), the order of linear part is determined using linear subspace identification method and the order of nonlinear part is based on the cross-validation technique. This separated order selection may not provide a compact Hammerstein model since the term selection problem is not considered in Gómez and Baeyens (2004).

Because the number of possible model terms may be very large, thus it may lead to a very complex model and an ill-condition problem. In this study, we want to obtain a parsimonious model for control, which should be as simple as possible. In fact, many terms are redundant and only a small number of important terms are necessary to describe the system with a given accuracy. The term selection problem has been extensively studied for the linear regression model (e.g., Billings, Korenberg & Chen, 1988a,b; Haber & Unbehauen, 1990; Lind & Ljung, 2008; Piroddi & Spinelli, 2003). In particular, the orthogonal forward regression (OFR) (Billings et al., 1988a, 1988b) is a fast and effective algorithm to determine significant model terms among a candidate set. Here we will extend the OFR to the Hammerstein model identification.

In this study, a time/space separation-based Hammerstein modeling approach is developed for unknown nonlinear distributed parameter processes with the spatio-temporal output and the temporal input. Firstly, the Karhunen-Loève decomposition is used for the time/space separation, where a few dominant spatial basis functions are estimated from the spatio-temporal data and the low-dimensional temporal coefficients are obtained simultaneously. Secondly, a low-order and parsimonious Hammerstein model is identified from the low-dimensional temporal data to establish the system dynamics, where the compact or sparse model structure is determined by the orthogonal forward regression algorithm, and the parameters are estimated using the least squares method and the singular value decomposition. The proposed time/space separated Hammerstein model has significant approximation capability to many nonlinear distributed parameter systems. With this model, many control and optimization algorithms designed for traditional Hammerstein model can be extended to nonlinear distributed parameter processes. The difference with the previous work (Qi et al., 2009) is the following:

- Different Hammerstein models are used. In the current paper, the Karhunen-Loève method is used for the time/space separation of the DPS, where a traditional Hammerstein is only used to model the temporal dynamics. In the previous paper, a new Hammerstein model is constructed with a Green's function (time/space nature) and a static nonlinear function. This new constructed Hammerstein model has time/space nature and is used to directly model DPS.
- Different identification algorithms are used. In the current paper, both model structure design and parameter estimation are considered with the OFR and the LSE–SVD algorithm, while the previous one only focuses on the parameter estimation with the LSE–SVD algorithm.

The main features of the proposed modeling method are: (1) the time/space separated Hammerstein model has a simple block-oriented nonlinear structure. (2) Owing to the Karhunen-Loève