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Numerical solution approaches for robust nonlinear optimal control problems

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Abstract

Nonlinear equality and inequality constrained optimization problems with uncertain parameters can be addressed by a robust worst-case formulation that leads to a bi-level min–max optimization problem. We propose and investigate a numerical method to solve this min–max optimization problem exactly in the case that the underlying maximization problem always has its solution on the boundary of the uncertainty set. This is an adoption of the local reduction approach used to solve generalized semi-infinite programs. The approach formulates an equilibrium constraint employing first order derivatives of both the uncertainty set and the user defined constraints. We propose two different ways for computation of these derivatives, one similar to the forward mode, the other similar to the reverse mode of automatic differentiation. We show the equivalence of the proposed approach to a method based on geometric considerations that was recently developed by some of the authors. We show how to generalize the techniques to optimal control problems. The robust dynamic optimization of a batch distillation illustrates that both techniques are numerically efficient and able to overcome the inexactness of another recently proposed numerical approach to address uncertainty in optimal control problems.
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1. Introduction

We consider uncertain nonlinear programming problems (NLP) of the form

$$\min_{x\in\mathbb{R}^{n_x},u\in\mathbb{R}^{n_u}} f_0(x,u,p),\quad \text{s.t.}\begin{cases} f(x,u,p)\le 0\\ g(x,u,p)=0\end{cases} \tag{1}$$

with uncertain parameters $p\in\mathbb{R}^{n_p}$. The optimization variables are partitioned into states $x\in\mathbb{R}^{n_x}$ and controls $u\in\mathbb{R}^{n_u}$. The objective function f_0, inequality constraints f, and equality constraints g are smooth functions which map from $\mathbb{R}^{n_p}\times\mathbb{R}^{n_x}\times\mathbb{R}^{n_u}$ into $\mathbb{R}$, $\mathbb{R}^{n_f}$, and $\mathbb{R}^{n_x}$, respectively. We assume the Jacobian $\partial g/\partial x$ to be invertible everywhere, so that we can regard the state variables x as an implicit function of u and p, which we will denote by $x(u,p)$. This division into states x and controls u arises naturally in model based optimization, where the equalities $g(x,u,p)=0$ often contain discretized ordinary or partial differential equations, such that we are in particular interested in case where $n_x\gg 1$.

We assume we have some knowledge about the uncertain parameters p such that they are restricted to the compact set

$$\mathbb{P}(u):=\{p\in\mathbb{R}^{n_p}|\exists x\in\mathbb{R}^{n_x}: g(x,u,p)=0, h(x,u,p)\le 0\}. \tag{2}$$

The nonlinear program (1) together with the uncertainty set (2) is a generalized semi-infinite program (GSIP) (Hettich & Kortanek, 1993) with differentiable functions h that map from $\mathbb{R}^{n_p}\times\mathbb{R}^{n_x}\times\mathbb{R}^{n_u}$ into $\mathbb{R}^q$. In this work we consider a subclass of GSIP with the assumption that h is a differentiable scalar function. This definition of the uncertainty set in particular includes the case of a confidence ellipsoid $\mathbb{P}_{\text{ellpsd.}}$ for a Gaussian random variable p with expectation value $\bar{p}$, variance–covariance matrix

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Σ, and a scalar $\gamma > 0$ depending on the desired confidence level (that is independent of u):

$$\mathbb{P}_{\text{ellpsd.}} = \{p \in \mathbb{R}^{n_p} | (p - \bar{p})^{\mathrm{T}} \Sigma^{-1} (p - \bar{p}) - \gamma \leq 0\}.$$

Other types of smooth uncertainty sets used in robust optimization also include confidence regions derived from the likelihood ratio test (Rooney & Biegler, 2001).

In order to incorporate the uncertainty in the optimization problem formulation, we choose the worst-case min–max formulation of (1). For this aim we assume that the optimizer, that chooses u first, has "nature" as an adverse player that chooses afterwards p and x. Whatever u the optimizer chooses, for each of the functions $f_i(x, u, p), i = 0, \ldots, n_f$, the worst case, $\phi_i(u)$, is chosen by the adverse player by selecting a suitable $p \in \mathbb{P}$:

$$\text{(WC)} \quad \begin{aligned} \phi_i(u) &:= \max_{p \in \mathbb{P}(u)} f_i(x(u,p), u, p) \\ &= \max_{x \in \mathbb{R}^{n_x}, p \in \mathbb{R}^{n_p}} f_i(x, u, p) \quad \text{s.t.} \quad \begin{cases} g(x, u, p) = 0, \\ h(x, u, p) \leq 0. \end{cases} \end{aligned} \tag{3}$$

Note that the adverse player "nature" is restricted by both the model equations $g(x, u, p) = 0$ and the scalar inequality constraint $h(x, u, p) \leq 0$. Employing the functions $\phi_i(u)$ we arrive at the following worst-case formulation that is often referred to as the "robust counterpart" of (1) (cf. Ben-Tal & Nemirovskii, 2001):

$$\text{(RC)} \quad \min_{u \in \mathbb{R}^{n_u}} \phi_0(u) \quad \text{s.t.}\ \phi_i(u) \leq 0; \quad \text{for} \quad i = 1, \ldots, n_f. \tag{4}$$

Due to the bi-level structure and the semi-infinite character, both formulations (1) and (RC) pose challenges for their efficient numerical solution. Different approaches to tackle these problems have been presented in a large number of articles. Methods to solve GSIP include discretization of the uncertainty set $\mathbb{P}(u)$ and the so called local reduction approach (Hettich & Kortanek, 1993). In both approaches the GSIP is approximated by a NLP with a finite number of constraints. In discretization methods problem (1) is solved on a finite grid of points $\bar{\mathbb{P}}(u) \subset \mathbb{P}(u)$ within the uncertainty set. The local reduction approach is based on the worst-case min–max formulation (RC). In this approach the constraints are reduced to a finite number of restrictions by considering only the (local) solutions of the inner maximization problem (WC), which are implicitly defined by the necessary conditions of optimality of (WC). In the locally reduced problem of (RC) then a finite number of worst cases within $\mathbb{P}(u)$ is tracked. For a series of comprehensive review articles on different solution strategies of GSIP-problems, including discretization and local reduction methods we refer to the recent monograph (Reemtsen & Rückmann, 1998).

Approaches to problem (1) have also been addressed by a number of articles on flexible and feasible process design. Measures of feasibility and flexibility have been introduced by Halemane and Grossmann (1983) and Swaney and Grossmann (1985). In this approach the optimization variables are partitioned into design and control variables. The design variables are specified by the outer minimization problem 1. Flexibility and feasibility in the presence of parametric uncertainty is addressed by formulating a nested max–min–max constraint for the feasibility constraints f, where the control variables are chosen by the minimization problem and the uncertain parameters by the outer maximization problem. The flexibility and feasibility measures were the basis for the development of various robust optimization methods in a series of papers, e.g., a two level optimization approach (Bahri, Bandoni, & Romagnoli, 1996) or an active set strategy (Mohideen, Perkins, & Pistikopoulos, 1996), which also considers worst-case dynamic disturbances. A different approach to robust optimization not based on the feasibility and flexibility measures was developed in a series of papers by Mönnigmann and Marquardt (2002, 2003, 2005). In this approach parametric uncertainty is taken into account by backing off the optimal design from critical boundaries in the space of the uncertain parameters. It furthermore allows the simultaneous treatment of feasibility and stability.

In this work we adopt the local reduction approach and present a numerical approach for the solution of (RC). We reformulate (RC) by introducing the necessary conditions of optimality of the inner maximization problem (WC). Due to the local character of the necessary conditions of optimality several worst-case points may exist for each function f_i, $i = 0, \ldots, n_f$. Afterwards we simplify the resulting constraints with the assumption that the worst-case point is always located on the *boundary* of the uncertainty set $\mathbb{P}(u)$. Solving the resulting NLP is then equivalent to track worst-case solutions situated on the boundary of the uncertainty set $\mathbb{P}(u)$. The local reduction approach is an efficient approach to rigorously solve (RC) also in case of a robust optimal control problem by tracking worst-case trajectories.

The aim in robust optimal control is to find an optimal profile of the manipulated variables such that none of the specified constraints are violated despite model uncertainties. Robust optimal control is also addressed by robust nonlinear model predictive control (NMPC). NMPC solves on-line repeatedly a dynamic optimization problem on a shrinking time horizon (Diehl, Bock, & Schlöder, 2005; Nagy & Braatz, 2003; Terwiesch, Agarwal, & Rippin, 1994) or on a moving horizon (Biegler, 2000; Binder et al., 2001; Diehl et al., 2002). For an assessment of the state of the art in this very active field we refer to Findeisen, Allgöwer, and Biegler (2006). Robust optimal control and robust NMPC is approached by Ma and Braatz (2001), Nagy and Braatz (2003), and Diehl, Bock, and Kostina (2006). For robust optimization, a linear or higher order approximation of the uncertain objective and constraints is used to faciliate the solution of the robust counterpart NLP (RC), involving some approximation errors. The algorithms of Ma and Braatz (2001) and Nagy and Braatz (2003) contain comparisons with full nonlinear uncertainty simulations. However, there it is not indicated how to numerically solve the exact robust counterpart NLP efficiently.

We have to mention here that the minimax approach for robust open-loop optimization is often considered to be too conservative to be useful in practice, as discussed, e.g., by Morari (1983). On the other hand the minimax approach allows to rigorously quantify the profit loss that has to be paid for the robustification

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