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Technical Note

THEORETICAL STUDY OF MOTION OF SMALL SPHERICAL AIR BUBBLES IN A UNIFORM SHEAR FLOW OF WATER

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ABSTRACT

A simple Couette flow velocity profile with an appropriate correlation for the free terminal rise velocity of a single bubble in a quiescent liquid can produce reliable results for the trajectories of small spherical air bubbles in a low-viscosity liquid (water) provided the liquid remains under uniform shear flow. Comparison of the model adopted in this paper with published results has been accomplished. Based on this study it has also been found that the lift coefficient in water is higher than its typical value in a high-viscosity liquid and therefore a modified correlation for the lift coefficient in a uniform shear flow of water within the regime of the Eötvös number $0.305 \leq Eo \leq 1.22$ is also presented.

1. Introduction

Water, being the most important working fluid in industrial processes, bears a great importance. Air–water two-phase flow systems are of interest for instance in chemical reactors, sparging, and mixing processes, heat transfer phenomena in a power plant's auxiliary systems, and nuclear power plants. It has been reported that the motion of small air bubbles inside flow channels is responsible for the reduction and/or increase of wall friction [1]. Therefore a substantial amount of research (although mostly related with high viscosity liquids) has been carried out into the behavior of air bubbles in quiescent or moving liquids [2–6]. These studies were based on either extensive computer computations or else complicated experiments.

It was reported by Celata et al. [7] that when an air bubble starts to rise inside a quiescent liquid its motion may fall into the viscous regime, the surface tension dominated regime or the inertia dominated regime depending on the dimensionless numbers that govern the physics. One such number is the Eötvös number that highlights the relative effect of buoyancy and surface tension forces. According to Clift et al. [8], at least for air–water systems the surface tension dominated regime holds when $0.25 < Eo < 40$. Many

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correlations depending upon the degree of contamination of the system exist in the literature that can handle all of the above regimes simultaneously [2,4,9,10]. However, since most of these correlations were developed for viscous liquids under stagnant conditions they need some tuning parameters in order to match the calculated trajectories with the experimental or simulated data if the problem involves a low viscosity liquid and a liquid velocity field as well. This particular fact has also been mentioned by Tomiyama et al. [4,11].

Therefore in this study we present a simple one step method (noniterative) that can be used to generate the bubble trajectories for small spherical bubbles (1.5 mm $\le$ d $\le$ mm) in a steady, laminar, fully developed, and uniform shear flow of water. It is shown in this study that a simple Couette flow velocity profile in combination with a general correlation for the free terminal rise velocity of the bubble in any stagnant Newtonian liquid proposed by Rodrigue [6] is sufficient to produce reliable results for the trajectories of small air bubbles. The properties of water and other relevant nondimensional numbers used in this study are shown in Table 1.

In this study we have considered a uniform shear rate of 6.2 s^{-1} for the sake of comparison and also to keep the flow under the laminar regime. It is well known that compared to other common liquids water has the highest surface tension coefficient with air and therefore within the regime of the Eötvös number considered in this study it is anticipated that there exist negligible changes in the shape of the bubbles. Therefore the bubbles are considered to remain spherical during their motion [3]. Another interesting and distinguishing feature of sheared liquids is that the motion of air bubbles having a diameter of up to 5.2 mm were observed to follow a trajectory without any zigzag motion [3,11,12]. This characteristic of a sheared liquid remains favorable for this work since any zigzag motion is avoided. Therefore, only those bubble diameters that can maintain straight trajectories without any significant size effects, which ultimately circumvent the use of the modified Eötvös number throughout this study, are considered. Compared to the above discussion, the trajectories of small spherical air bubbles > 1.5 mm in diameter were observed to follow a zigzag pattern in the case of stagnant liquids [13–15].

2. Mathematical formulations

In order to proceed, we first need to establish the expressions for the components of the bubble relative velocity in the horizontal and vertical directions. It is to be noted that in this study the bubble motion remains in two dimensions, mainly due to the range of the Eötvös number considered here. The geometry of the problem is depicted in Fig. 1, which represents the schematic of a simple Couette flow. The liquid (water) between the moving and the stationary plate remains under steady, laminar, fully developed, and uniform shear flow. The width between the plates is W = 30 mm and the origin of the coordinate system (L) coincides with the point of injection of the bubble and can be adjusted to arbitrary locations. Finally, U is the velocity of the moving plate. The general equation of motion of a bubble inside any geometrical domain can be expressed as Newton's second law of motion in Lagrangian coordinates by Equation 1 [3,4].

$$\left(\rho_g + 0.5\rho_l\right)\frac{DV_b}{Dt} = -\frac{3}{4}\frac{C_D}{d}\rho_l\left|V_r\right|V_r - C_L\rho_l V_r \times \nabla \times V_r + \left(\rho_l - \rho_g\right)g \quad (1)$$

In Equation 1 V_r is the bubble relative velocity, V_l is the liquid velocity, V_b is the bubble velocity, ρ_g and ρ_l represent the density of the gaseous phase (air) and liquid phase (water), respectively, d is the diameter of the bubble, C_D and C_L are the drag and lift coefficients respectively, and g is the acceleration due to gravity. The first term on the right hand side of Equation 1 represents the drag force, the second term represents the lift force, and the last term represents the effect of gravity, the so-called buoyancy force. It must be noted that the wall effects can be considered to be negligible in this study. Therefore following Rahba and Buwa [3] and Bothe et al. [5] Equation 1 can be split into its horizontal and vertical components as follows.

2.1. *Horizontal component of bubble equation of motion*

$$0.5\frac{dV_{rx}}{dt} = -\frac{3}{4}\frac{C_D}{d}\left|V_r\right|V_{rx} + C_L\xi V_{ry} \quad (2)$$

The horizontal component of Equation 1 is therefore given by Equation 2. In Equation 2 V_{rx} is the relative bubble velocity in the lateral or horizontal direction, V_{ry} is the relative bubble velocity in the vertical direction, and ξ is the magnitude of the

Table 1 – Fluid properties and dimensionless numbers used in this study.

Properties/ dimensionless no.	Water	Air bubble
$\rho(kg/m^3)$	998	1.2
$\mu(Pa.s)$	0.00098	—
$\sigma(N/m)$ for air–water interface	0.072	—
Eo	0.305–1.22	—
M	2.4252 $\times 10^{11}$	—

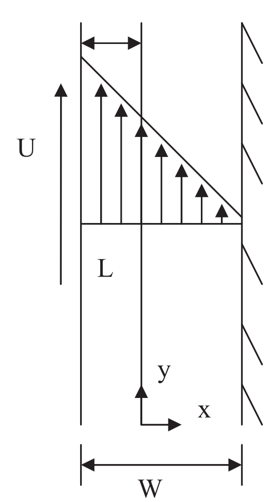

Fig. 1 – Geometry and coordinate system for the problem domain.