Progress in Nuclear Energy 82 (2015) 102–106

Contents lists available at ScienceDirect

Progress in Nuclear Energy

journal homepage: www.elsevier.com/locate/pnucene

Photonuclear reactions of calcium isotopes calculated with the nuclear shell model

Yutaka Utsuno [a, b, *], Noritaka Shimizu [b], Takaharu Otsuka [b, c, d], Shuichiro Ebata [e], Michio Honma [f]

[a] Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan
[b] Center for Nuclear Study, University of Tokyo, Hongo, Tokyo 113-0033, Japan
[c] Department of Physics, University of Tokyo, Hongo, Tokyo 113-0033, Japan
[d] Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA
[e] Meme Media Laboratory, Hokkaido University, Sapporo-shi 060-0810, Japan
[f] Center for Mathematical Sciences, University of Aizu, Ikki-machi, Aizu-Wakamatsu, Fukushima 965-8580, Japan

A R T I C L E I N F O

Article history:
Received 29 October 2013
Received in revised form
30 January 2014
Accepted 24 July 2014
Available online 18 August 2014

Keywords:
Photoabsorption cross section
Shell model
Giant dipole resonance
Pygmy dipole resonance
*E*1 transition
Lanczos method

A B S T R A C T

Photoabsorption cross sections in calcium isotopes are investigated theoretically with the most advanced nuclear-structure calculations using the nuclear shell model. We show that the photoabsorption cross sections in ^{48}Ca dominated by a giant dipole resonance (GDR) are well reproduced with the shell model and predict that a pygmy dipole resonance (PDR) appears in ^{52}Ca in the low-energy tail of GDR. Excellent agreement with the experiment is achieved due to highly correlated many-body wave functions obtained by treating a huge matrix with the dimension of ~10^{10}.

1. Introduction

Since photons (γ rays) are usually emitted during nuclear reactions including fissions, the photonuclear reaction constitutes one of the fundamental reaction processes which take place in nuclear reactors, accelerators, and other environments involving radioactivity. Moreover, the photonuclear cross section is closely related to the neutron-reaction cross section and provides a way to estimate it because a (γ, n) reaction is the inverse reaction of an (n, γ) reaction. Thus, photonuclear data are needed for a variety of applications, and are available in several nuclear-data libraries (Obložinský, 2002).

However, the photonuclear data measured so far are restricted to (semi-) stable nuclei, although the nuclei for radioactive (or unstable) isotopes also have potential use. For example, a photonuclear reaction is a possible choice for transmuting radioactive nuclides ^{90}Sr and ^{137}Cs which have neutron reaction cross sections too small to transmute with neutrons. Photonuclear reaction cross sections for unstable nuclei can be estimated roughly from systematics: a large peak called a giant dipole resonance (GDR) are located at around $80A^{-1/3}$ MeV (Ring and Schuck, 1980). A more detailed structure, which is of practical importance in some cases, however, depends on individual nuclei. For instance, in the low-energy side near the neutron threshold energy another small peak called a pygmy dipole resonance (PDR) appears in some nuclei, but which nuclei have the PDR and how large its peak is not clear. Reliable microscopic calculations, which can be carried out without regard to the stability of nuclei, are thus highly desired. While the random-phase approximation (RPA) has been developed and used for this purpose (for instance, see Inakura et al. (2009)), some deviations from experiments are seen particularly in light nuclei.

The aim of this paper is to demonstrate that most advanced nuclear-structure calculations with the nuclear shell model are able to excellently describe the photonuclear cross sections as exemplified by calcium isotopes. Since the shell-model calculations for the photonuclear cross section require much numerical

* Corresponding author. Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan.
E-mail address: utsuno.yutaka@jaea.go.jp (Y. Utsuno).

http://dx.doi.org/10.1016/j.pnucene.2014.07.036
0149-1970/

computation, its application was rather limited to very light nuclei and doubly-magic nuclei (Sagawa and Suzuki, 1999; Schwengner et al., 2010), while an *ab initio* (i.e., first principles) approach based on the shell model has also been adopted (Barbieri et al., 2008). The present calculation is enabled by the development of a shell-model code KSHELL (Shimizu, 2013) which runs on massively parallel supercomputers.

2. Theoretical framework

2.1. Photoabsorption cross section

Photonuclear reactions proceed within the following two steps: (i) a nucleus, in general located at its ground state, is excited to an excited state by absorbing a γ ray, and (ii) the state excited by the γ ray is deexcited by emitting particles such as neutrons and γ rays. In the present study, we discuss only the absorption process (i), whose cross section is referred to as a photoabsorption cross section and consists of the cross sections of (γ,γ'), (γ,n), $(\gamma,2n)$, ... reactions. Since the photoabsorption reaction occurs as an electromagnetic process, its formulation is established. For low-energy photons with $ER/(\hbar c)\ll 1$, where E and R are the energy of the γ ray and the radius of a nucleus, respectively, the $E1$ transition dominates the photoabsorption process, and therefore we calculate only the $E1$ transitions in this study. The cross section for exciting from the ground state $|0\rangle$ to an excited state $|\nu\rangle$ with an incoming photon polarized in the z direction via the $E1$ transition is described (Ring and Schuck, 1980) as

$$\sigma_\nu(E) = \frac{4\pi^2 e^2}{\hbar c}(E_\nu - E_0)\left|\langle\nu|D_z|0\rangle\right|^2\delta(E - E_\nu + E_0), \tag{1}$$

where E_ν and E_0 are the energies of the $|\nu\rangle$ and the $|0\rangle$ states, respectively, and D_z stands for the dipole operator in the z direction:

$$D_z = \frac{N}{A}\sum_{p=1}^{Z} z_p - \frac{Z}{A}\sum_{n=1}^{N} z_n. \tag{2}$$

Here, z_p and z_n are one-body operators for protons and neutrons, respectively. The photoabsorption cross section is obtained by summing over the excited states $|\nu\rangle$ as

$$\sigma_{\rm abs}(E) = \sum_\nu \sigma_\nu(E). \tag{3}$$

In the usual case, where the photon and the target nucleus are unpolarized, the z dependence in Eq. (1) can be removed and the resulting photoabsorption cross section becomes

$$\sigma_{\rm abs}(E) = \frac{16\pi^3 E}{9\hbar c} S_{E1}(E), \tag{4}$$

where $S_{E1}(E)$, often denoted as $\mathrm{d}B(E1;E)/\mathrm{d}E$, is called the $E1$ strength function given by

$$S_{E1}(E) = \sum_\nu B(E1;\mathrm{g.s.}\to\nu)\delta(E - E_\nu + E_0) \tag{5}$$

with the $B(E1)$ value from the ground state.

2.2. Nuclear shell model

One can see from Eq. (1) that obtaining good many-body wave functions $|0\rangle$ and $|\nu\rangle$ is the most essential task for calculating accurate photoabsorption cross sections. In the present study, many-body wave functions are calculated with the nuclear shell model. Based on the independent particle picture, the shell model takes into account limited single-particle degrees of freedom to construct many-body states. For instance, when one describes the ground state of a calcium isotope with $A > 40$, the objective of this study, it is often assumed that 20 protons and 20 neutrons among A nucleons form an inert core by fully occupying the orbits belonging to the $\mathcal{N} = 0, 1, 2$ major shells, where the major shell is classified according to the eigenvalue of the harmonic-oscillator single-particle Hamiltonian taking $E(\mathcal{N}) = (\mathcal{N} + 3/2)\hbar\omega$. This restriction is a good approximation, because exciting a nucleon to the next harmonic oscillator shell needs as much energy as $1\hbar\omega \sim 10$ MeV and thus results in a small component in the ground state. The remaining $A - 40$ nucleons are regarded as valence nucleons which partly occupy the $\mathcal{N} = 3$ major shell consisting of the $0f_{7/2}$, $0f_{5/2}$, $1p_{3/2}$ and $1p_{1/2}$ valence orbits. The many-body wave function characterized by the occupancy of the valence nucleons is expressed as a linear combination of many-body basis states consisting of every possible occupation of the valence orbits by the valence nucleons. The coefficients of the linear combination are determined by diagonalizing a many-body Hamiltonian matrix spanned by the many-body basis states whose matrix elements are calculated with a given effective nucleon–nucleon interaction.

The $E1$ excited states $|\nu\rangle$ require active valence orbits more than the orbits needed for describing the ground state. The $E1$ transition involves parity change, but the opposite parity states are not available within a single major shell which includes only orbits with the same parity. Thus, excitation across the major shells must be taken into account. For the present purpose, one should include many-body states in which a single-nucleon excitation occurs from the $\mathcal{N} = 2$ major shell to the $\mathcal{N} = 3$ one and also from the $\mathcal{N} = 3$ major shell to the $\mathcal{N} = 4$ one.

Following the above discussion, we take the $\mathcal{N} = 2, 3, 4$ major shells as the valence shell to calculate photoabsorption cross sections in calcium isotopes $^{42-58}$Ca, and appropriately restrict the number of nucleons excited across the major shells. The nucleon excitation can be effectively controlled by classifying many-body states according to the total harmonic-oscillator quantum number $\sum_k \mathcal{N}_k$ measured from the lowest value for a given nucleus. For instance, many-body states with the $A - 40$ valence nucleons

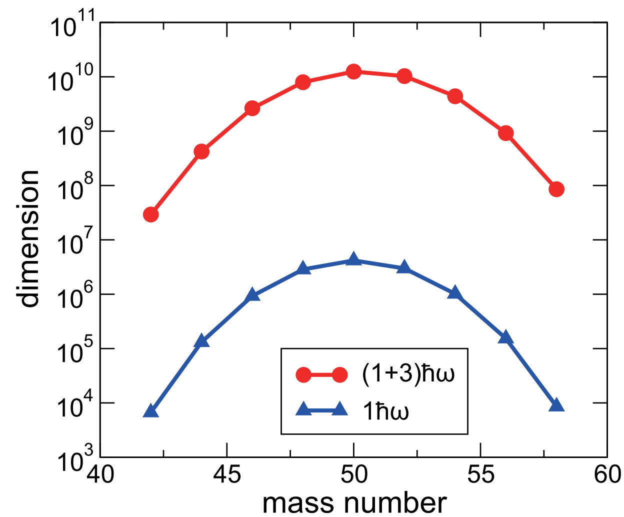

Fig. 1. Dimensions of the shell-model Hamiltonian matrices for the $1\hbar\omega$ and $(1+3)\hbar\omega$ calculations in $^{42-58}$Ca. The numbers of basis states with $J_z = 1$ and $\pi = -1$ (called M-scheme dimension) are plotted, where J_z and π denote the z component of the total angular momentum and parity quantum numbers, respectively.