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Model and analysis of performance for the method of characteristics direction probabilities with boundary averaging

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The work presents a performance model of the method of characteristic direction probabilities (CDP) which integrates the benefits of the collision probability method (CPM) and the method of characteristics (MOC) for solution of the integral form of the Boltzmann Transport Equation and has been implemented in the Michigan PArallel Characteristic based Transport (MPACT) code for 2-D and 3-D transport calculations. The process of boundary averaging reduced the storage and computation but the capability of dealing with complicated geometries is preserved since the same ray tracing information is used as in MOC. The benefits of CDP are demonstrated by the developed performance model which describes the storage, floating point operations and calculation time. The numerical results are given for different cases to show the accuracy, storage, floating point operations and computing time of the CDP compared to the MOC using the performance model. From the cases examined, the boundary average method shows significant improvement on the storage and computational efficiency for three-dimensional cases with sufficient accuracy.

1. Introduction

The method of characteristics (MOC) based on the modular ray tracing technique (Liu et al., 2011) has been implemented in Michigan PArallel Characteristic based Transport code (MPACT) to perform lattice and whole core calculations for LWR applications. The MOC uses a set of discrete ordinates, which is similar to the S_N methods, but MOC is better suited to treat complicated geometries because it only requires an approximation on the spatial variation of the source, and not on the flux itself along the rays. However, the transport sweep needs to be performed along all the characteristics lines for every direction, and this sweeping time will be computationally expensive when calculating three-dimensional problems, in which the number of characteristic rays could be quite large to accurately present very thin regions using burnable absorbers such as IFBA that coat the fuel pin.

In the previous work (Liu et al., 2013), the method of characteristic direction probabilities, which was first proposed by (Hong and Cho, 1999), was implemented in MPACT to minimize the computation efforts. This new transport method couples the desirable features of the MOC and the collision probability (CPM) (Sanchez, 1997). CPM has been widely used in lattice physics codes because it has the capability of treating the complicated geometries and is very efficient when dealing with small size problems. But this method has the drawback that the storage requirements and computing time depend on the square of the number of fine spatial regions in the problem. This is because the collision probability matrix couples all the fine mesh regions. To overcome the drawback of CPM when dealing with big size problems, the interface current method (ICM) (Mohanakrishnan, 1981) was developed which couples the sub-domains with interface current of interface current moments, and within the domains the fine regions are coupled by the CPM. However, compared to the interface current method, CDP doesn't introduce the approximation at the interface of the sub-domain and the anisotropic sources. Another drawback of CPM is that it cannot easily treat anisotropic sources. In the CDP, only fine regions traversed by a characteristic line within a specified sub-domain are coupled which is the most significant difference with

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CPM. At the same time, the CDP is capable of providing the same accuracy as MOC if the unique boundary sub-domains are the same size as the MOC ray spacing. The only difference in the methods then would be that instead of performing the transport sweep ray by ray of the MOC, the CDP method obtains the outgoing angular flux and fine region flux by direct multiplication of a matrix which contains the collision and transmission probabilities and a vector which includes the coming angular flux and the fine region source. The collision and transmission probabilities in the CDP are derived by integrating the traditional MOC equations along a characteristic line. So in principle, the method of characteristics direction probabilities is mathematically consistent with the conventional MOC. To further improve the efficiency of the CDP, the boundary averaged ray tracing technique was introduced which can reduce the memory required for storing the probabilities and improve computing efficiency.

A performance model is described in this work to explicitly analyze the storage requirement, floating point operations and computing time. Based on the analysis, we can found where and how much the boundary-averaged CDP earns the profit. The examined numerical results proved the consistence of the performance model to the measured results and showed the advantage of the boundary-averaged CDP.

In the following section the basic equations of the MOC are provided along with the derivation of the CDP method. Also described in this section is boundary average scheme. The third section introduces the performance model of the CDP and numerical results are shown in the subsequent section. The final section provides a summary and conclusions.

2. The method of characteristics direction probabilities

2.1. The method of characteristics

The classical method of characteristics for solving partial differential equations has been successfully applied to the Boltzmann Transport equation (BTE) and implemented in several reactor analysis codes. The group-wise form of the BTE for the system R is given by.

$$\Omega\cdot\nabla\varphi_g(\mathbf{r},\mathbf{\Omega})+\Sigma_{t,g}\varphi_g(\mathbf{r},\mathbf{\Omega})=Q_g(\mathbf{r},\Omega), \tag{1}$$

where $Q_g(\mathbf{r})$ is total source including both the fission source term and the scattering source terms.

The MOC equation provides a solution of the Boltzmann Transport equation along a line in a particular direction and it reduces to the total differential Equation (2) which is simplified by removing the energy group subscript g.

$$\begin{aligned}&\frac{d\varphi(\mathbf{r}_0+s\mathbf{\Omega}_m,\mathbf{\Omega}_m)}{ds}+\Sigma(\mathbf{r}_0+s\mathbf{\Omega}_m)\varphi(\mathbf{r}_0+s\mathbf{\Omega}_m,\mathbf{\Omega}_m)\\&\quad=Q(\mathbf{r}_0+s\mathbf{\Omega}_m,\mathbf{\Omega}_m),\end{aligned} \tag{2}$$

where $\mathbf{r}_0$ is the starting point of a characteristic line and s is the distance from the initial point to the current point along a specified direction $\mathbf{\Omega}_m$.

When solving the equation, we assume that the source and properties are constant in a small region D_i.

$$Q(\mathbf{r},\mathbf{\Omega}_m)=Q_i(\mathbf{\Omega}_m),\quad \Sigma_t(\mathbf{r}_0+s\mathbf{\Omega}_m)=\Sigma_{t,i},\quad \mathbf{r}\in D_i \tag{3}$$

In this small region if we know the incoming angular flux along the line k which starts at the boundary and which can be written as $\varphi_{i,k}^{in}(\mathbf{\Omega}_m)$, then outgoing angular flux from D_i along the line can be calculated as:

$$\varphi_{i,k}^{out}(\mathbf{\Omega}_m)=\varphi_{i,k}^{in}(\mathbf{\Omega}_m)\exp(-\Sigma_{t,i}s_{i,k})+\frac{Q_{i,k}(\mathbf{\Omega}_m)}{\Sigma_{t,i}}[1-\exp(-\Sigma_{t,i}s_{i,k})], \tag{4}$$

where $s_{i,k}$ is the length between the outgoing point and the incoming point of the line k in D_i.

The average segment angular flux can then be given as:

$$\overline{\varphi}_{i,k}(\mathbf{\Omega}_m)\cdot s_{i,k}=\frac{Q_{i,k}(\mathbf{\Omega}_m)}{\Sigma_{t,i}}s_{i,k}+\frac{\varphi_{i,k}^{in}(\mathbf{\Omega}_m)-\varphi_{i,k}^{out}(\mathbf{\Omega}_m)}{\Sigma_{t,i}} \tag{5}$$

2.2. Method of characteristics direction probabilities

The three-dimensional characteristics direction probabilities (CDP) include the directional transmission and collision probabilities which are stored for all the unique geometries of the problem. The transmission and collision probabilities are derived by integrating the MOC equation from the incoming boundary to the outgoing boundary. For a given geometry sub-domain (see Fig. 1), the outgoing angular flux of the sub-boundary can be written in terms of the probabilities as:

$$\begin{aligned}\varphi_{bo}^{out}(\mathbf{\Omega}_m)=&\sum_{bi\in N(bo)}T_{bi->bo}(\mathbf{\Omega}_m)\varphi_{bi}^{in}(\mathbf{\Omega}_m)\\&+\sum_{j\in J(bo)}T_{j->out}(\mathbf{\Omega}_m)Q_j(\mathbf{\Omega}_m)\end{aligned} \tag{6}$$

where

$$T_{bi->bo}(\mathbf{\Omega}_m)=\sum_{k\in(bo\cap bi)}\frac{A_k\exp\left(-\sum_{j=1}^{i}\left(\Sigma_{t,j}s_{j,k}\right)\right)}{A_{bo}} \tag{7}$$

$$T_{j->bo}(\mathbf{\Omega}_m)=\sum_{k\in(bo\cap j)}\frac{A_k\left[1-\exp\left(-\Sigma_{t,j}s_{j,k}\right)\right]\exp\left(-\sum_{l=j+1}^{i}\Sigma_{t,l}s_{l,k}\right)}{A_{bo}\Sigma_{t,j}} \tag{8}$$

where $\varphi_{bi}^{in}(\mathbf{\Omega}_m)$ and $\varphi_{bo}^{out}(\mathbf{\Omega}_m)$ represent the incoming angular flux and outgoing angular flux of the sub-boundary, respectively, and where k is the characteristic line index and i is the flat source region index along the characteristic line shown in Fig. 1, and where $j=1$ is the first region traversed by the characteristic line k, and where A_k is the cross section area of the characteristic track orthogonal to the characteristic track direction, A_{bo} is the projection area of the sub-boundary, and $k\in(bo\cap bi)$ means the characteristic lines traverse both the bi and bo sub-boundaries.

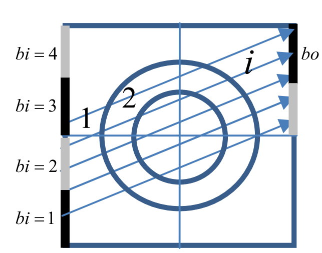

Fig. 1. The modular geometry sub-domain.