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A novel mathematical model for two-energy groups of the point kinetics reactor dynamics

Ahmed E. Aboanber [a, b, *], Abdallah A. Nahla [a], Zeid I. Al-Muhiameed [b]

[a] Department of Mathematics, Faculty of Science, Tanta University, Tanta 31527, Egypt
[b] Department of Mathematics, Faculty of Science, Qassim University, Buraidah, Saudi Arabia

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ABSTRACT

The point kinetics equations for reactor dynamic systems are normally described and treated for one-energy group, which modeled as stiff coupled differential equations, and their solution by the conventional explicit methods will give a stable consistent result only for very small time steps. A novel analytical formulation is constructed and converged to high accuracy from the merger of the piecewise constant functions over a partition in time into the fundamental matrix for the two-energy group of the point kinetics equations. The resulting system of stiff linear and/or nonlinear differential equations for an arbitrary number of delayed neutrons is solved exactly over each time step. Through analytical inversion technique of the fundamental matrix and the stability of the method, we demonstrate its high accuracy for a variety of imposed reactivity insertions found in the literature for three dimensional homogeneous reactors. From knowledge of how the error term behaves the computational results indicate that the method is efficient and accurate for multi-dimensional homogeneous reactors.

1. Introduction

The safety of nuclear power reactors has become an urgent requisite for all world countries, which implies a physically powerful effort to arrive to an efficient and economic worldwide mathematical development codes for nuclear criticality. According to the fact that, there are a large number of fission product isotopes that decay by neutron emission and, thus, are members of the delayed precursor family. For the purposes of modeling their effect on neutron kinetics, it is sufficient to group them into six groups according to their half-life. A fast and an accurate algorithm for modeling and solving the time dependant neutron diffusion equations for reactor kinetics with six groups of delayed neutrons is developed in this work to improve the safety of nuclear power reactors. The algorithm is derived by means of analytical and numerical mathematical tools. This solution describes and predicts the temporal evolution of the neutron flux and precursor concentrations of delayed neutrons for the multi-energy point kinetics reactor systems.

Several estimated solutions of the neutron dynamics in the reactor cores have been analyzed and reviewed by many workers (e.g. Dahmani et al., 2001; Kobayashi, 2005; Gupta et al., 2005; Grossman and Hennart, 2007; Ginestar et al., 2002; Miró et al., 2002; González-Pintor et al., 2010; Tamitani et al., 2003; Shimjith et al., 2010; Aboanber and Hamada, 2008, 2009; Quintero-Leyva, 2010). General class of alternating-direction semi-implicit methods, numerical code (3DKIN), was presented by Ferguson and Hansen (1973) for solving the space-dependent reactor kinetics equations in three dimensions. Adaptive matrix formation (AMF) method was developed by Aboanber and Nahla (2007) to solve the transient multi-groups neutron diffusion and delayed precursor equations in two- and three-dimensional geometry. This method offers the flexibility of using small time steps between flux shape calculations to achieve a specified accuracy and capability, without encountering numerical problems.

In the present work we elaborate a methodology to solve, and adopted analytically by the fundamental matrix and piecewise constant functions over a partition in time, the kinetic diffusion equation of two energy groups with six groups of delayed neutrons. In 1988 Blanchon et al. introduced a numerical method for simulating the neutron kinetics in a reactor with two energy groups and six precursor groups. The stability has been evaluated and the iterative algorithm was used for propose of the solution of a large linear system. Lemos et al. (2008) solved the diffusion equation of

* Corresponding author. Department of Mathematics, Faculty of Science, Tanta University, Tanta 31527, Egypt.
E-mail addresses: aaboanber@yahoo.com, aboanber@tu.edu.eg (A.E. Aboanber), nahla@tu.edu.eg (A.A. Nahla), ksapr006@yahoo.com (Z.I. Al-Muhiameed).

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neutrons in slab geometry for a model with two energy groups by the technique of Laplace transform. The problem in Cartesian geometry was solved successfully by Ceolin et al. (2010) and was extended for a different geometry. Recently, Fernandes et al. (2011) introduced an analytical solution for the two-group kinetic neutron diffusion equations in cylindrical geometry by the Hankel transform. Furthermore in another work, Fernandes et al. discussed, in (2013), the kinetic neutron diffusion equation in homogeneous cylinder geometry. They construct solutions unaffected by a numerical artifact, known as the stiffness of the equation system, for two energy groups, one and six precursor concentrations, respectively.

In this work we focus on the derivation of an analytical formulation for the system of differential equations representing the neutron flux for fast and thermal energy groups and the contributions from the precursors. The principal idea is to solve the point kinetics diffusion equation of neutrons, for the model of two energy groups using the fundamental matrix method whereas the stiffness of the system is treated by means of the piecewise constant functions over a partition in time. To this end, the eigenvalues of the coefficient matrix are calculated numerically using FORTRAN computer code based on Laguerre's method. Also, The eigenvectors of the coefficient matrix and the inverse of the fundamental matrix are calculated analytically. Finally, a generalized system involving two groups of equations, two for flux, fast and thermal, and six of kinetic origin, representing six different groups of delayed neutrons is obtained. Furthermore, the stability of the developed methodology is discussed and in view of the promising results in this work the results are compared well with the conventional methods.

2. Mathematical model

The deterministic time-dependent equations satisfied by the neutron flux and the delayed neutron precursors can be described by the two energy groups neutron diffusion equations with I-groups of delayed neutron precursors (Hetrick, 1993; Glasstone and Sesonske, 1994; Stacey, 2001; Aboanber and Nahla, 2006, 2007):

$$\frac{1}{v_1}\frac{\partial}{\partial t}\Phi_1(r,t) = D_1\nabla^2\Phi_1(r,t) - (\Sigma_{a_1}+\Sigma_{s_{12}})\Phi_1(r,t) + \nu\Sigma_{f_1}(1-\beta)\Phi_1(r,t) + \nu\Sigma_{f_2}(1-\beta)\Phi_2(r,t) + \sum_{i=1}^{I}\lambda_i C_i(r,t) \quad (1)$$

$$\frac{1}{v_2}\frac{\partial}{\partial t}\Phi_2(r,t) = D_2\nabla^2\Phi_2(r,t) - \Sigma_{a_2}\Phi_2(r,t) + \Sigma_{s_{12}}\Phi_1(r,t) \quad (2)$$

$$\frac{\partial}{\partial t}C_i(r,t) = \beta_i\left(\nu\Sigma_{f_1}\Phi_1(r,t) + \nu\Sigma_{f_2}\Phi_2(r,t)\right) - \lambda_i C_i(r,t), \quad i = 1,2,\cdots,I \quad (3)$$

This model governing the dynamic groups diffusion neutron flux and delayed precursor concentration behavior, where $\Phi_1(r,t)$ and $\Phi_2(r,t)$ are fast and thermal neutron flux, $C_i(r,t)$ is the delayed precursor concentration, D_1 and D_2 are fast and thermal diffusion coefficients, Σ_{a_1} and Σ_{a_2} are fast and thermal absorption cross sections, Σ_{f_1} and Σ_{f_2} are fast and thermal fission cross sections, $\Sigma_{s_{12}}$ is the scattering cross section from fast to thermal neutron, ν is the neutron fission, v_1 and v_2 are fast and thermal neutron speed, λ_i is the decay constant of i-group of delayed neutrons and β_i is the fraction of i-group delayed neutrons.

The differential equations representing the neutron flux for two energy groups and precursor concentrations of delayed neutrons can be obtained from Equations (1)–(3) by separation of variables. These separated differential equations can then be used to solve the kinetics problem in general. Let us assume that:

$$\Phi_1(r,t) = v_1 n_1(t)\widehat{\Psi}(r), \quad \Phi_2(r,t) = v_2 n_2(t)\widehat{\Psi}(r), \quad C_i(r,t) = C_i(t)\widehat{\Psi}(r) \quad (4)$$

where, $n_1(t)$ and $n_2(t)$ are the fast and thermal neutron density, $C_i(t)$ is the precursor concentration density of delayed neutrons, and $\widehat{\Psi}(r)$ is the fundamental function which can be determined from the standard diffusion equation describing the flux shape in the reactor for both groups:

$$\nabla^2\widehat{\Psi}(r) + B^2\widehat{\Psi}(r) = 0 \quad (5)$$

where, B^2 is the material buckling given by:

$$B^2 = -\frac{\Sigma_{a_1}+\Sigma_{s_{12}}-\nu\Sigma_{f_1}}{2D_1} - \frac{\Sigma_{a_2}}{2D_2} + \sqrt{\left(\frac{\Sigma_{a_1}+\Sigma_{s_{12}}-\nu\Sigma_{f_1}}{2D_1}+\frac{\Sigma_{a_2}}{2D_2}\right)^2 - \left(\frac{\Sigma_{a_2}\left(\Sigma_{a_1}+\Sigma_{s_{12}}-\nu\Sigma_{f_1}\right)-\Sigma_{s_{12}}\nu\Sigma_{f_1}}{D_1D_2}\right)} \quad (6)$$

Substituting Equations (4) and (5) into Equations (1)–(3) yields

$$\frac{dn_1(t)}{dt} = -D_1B^2v_1n_1(t) - (\Sigma_{a_1}+\Sigma_{s_{12}})v_1n_1(t) + \nu\Sigma_{f_1}(1-\beta)v_1n_1(t) + \nu\Sigma_{f_2}(1-\beta)v_2n_2(t) + \sum_{i=1}^{I}\lambda_iC_i(t) \quad (7)$$

$$\frac{dn_2(t)}{dt} = -D_2B^2v_2n_2(t) - \Sigma_{a_2}v_2n_2(t) + \Sigma_{s_{12}}v_1n_1(t) \quad (8)$$

$$\frac{dC_i(t)}{dt} = \beta_i\left(\nu\Sigma_{f_1}v_1n_1(t) + \nu\Sigma_{f_2}v_2n_2(t)\right) - \lambda_iC_i(t), \quad i = 1,2,\cdots,I \quad (9)$$

Let us consider the following kinetic parameters, $l_1 = 1/v_1\nu\Sigma_{f_1}$ and $l_2 = 1/v_2\nu\Sigma_{f_2}$ are fast and thermal generation time between birth of neutron and subsequent absorption inducing fission, $L_1^2 = D_1/\Sigma_{a_1}$ and $L_2^2 = D_2/\Sigma_{a_2}$ are fast and thermal diffusion length, $k_1 = \nu\Sigma_{f_1}/\Sigma_{a_1}[1+L_1^2B^2]$ and $k_2 = \nu\Sigma_{f_2}/\Sigma_{a_2}[1+L_2^2B^2]$ are fast and thermal multiplication factor, $\rho_1 = k_1 - 1/k_1$ and $\rho_2 = k_2 - 1/k_2$ are fast and thermal reactivities and $\kappa = v_1\Sigma_{s_{12}}$.

Upon substituting into Equations (7)–(9) the following system is obtained:

$$\frac{dn_1(t)}{dt} = \left(\frac{\rho_1-\beta}{l_1}-\kappa\right)n_1(t) + \left(\frac{1-\beta}{l_2}\right)n_2(t) + \sum_{i=1}^{I}\lambda_iC_i(t) \quad (10)$$