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Hybrid-transport point kinetics for initially-critical multiplying systems

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ABSTRACT

The paper presents an extension of the Hybrid Transport Point Kinetic (HTPK) model to initially-critical multiplying structures, i.e. in a source-free medium with transients starting from the equilibrium between neutron distribution and precursor concentrations. The mathematical model behind the HTPK methodology is derived from the detailed time-dependent balance equations and with reference to the limiting cases (i.e., point model and multi-collision models). Numerical simulations of transient systems demonstrate the interesting features of HTPK, which is shown to sensibly increase the accuracy of classical point kinetics models even at low truncation orders.

1. Introduction

The study of neutron kinetics in multiplying systems such as nuclear reactors is generally approached with simplified methods due to its computational burden. A bookkeeping method developed in the past and widely used in applications is the so-called Point Kinetic method (PK). The model consists in a lumped parameter approximation of the reactor and precursor concentrations (Henry and Curlee, 1958). The point kinetic system of ODEs is derived by factorizing the solution (i.e., neutron flux) into shape and amplitude and subsequently projecting the original equation onto a suitable weight. In this projection process, the space, directional and energetic information is condensed into the coefficients for the system of first-order ODEs. Obviously, when significant spectral transients occur, this approximation becomes rather inefficient due to a strong evolution of the shape during the transient. In such cases, a possible generalization of PK is the Multi-Point Kinetics (MPK). Instead of a single amplitude for the whole neutron distribution, MPK models with several amplitudes different space/energy regions in the phase space (Kobayashi, 1991, 1992; Ravetto et al., 2004). Although the method is accurate when applied to loosely coupled reactors (Avery, 1958), the advantage of multi-point approach is drastically reduced when dealing with initially critical systems where the evolution of neutron density in different regions of phase space is much more connected (e.g., (Picca)). Another option often considered in nuclear applications is the Quasi Static method (QS), initially proposed by Ott and Meneley (1969) and then improved by Devooght (1980). In QS, the shape is allowed to vary on a coarser mesh to account for the spatial/energy features exhibited during the transient. Depending on the reactor physical configuration, the computational performance can significantly change due to the number of iterations for convergence on normalization condition (e.g., (Dulla et al., 2008)).

Recently, a novel method has been proposed (Picca et al., 2011, 2012) for modeling reactor kinetics in subcritical system. Conceived as an extension of the PK method, the Hybrid Transport Point Kinetics (HTPK), considers a direct simulation of the first generation of neutrons with multi-collision transport approach and applies the lumped parameter method to the residue after truncation of the multicollision series. In the above mentioned papers, the method is derived following the physical intuition that associates to source/first generation neutrons the major effect related to spectral transients and assumes the appearance of a bulk behavior after several collisions. For this reason, while HTPK is naturally suited for source-driven problems, its application to source-free systems needs some additional mathematical developments.

The purpose of the paper is to extend HTPK to deal with initially critical systems, taking inspiration from the well-known power

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iteration method used for converting an eigenvalue problem in the iterative solution of a source problem (Lewis and Miller, 1993). The contribution of precursor emissions, not considered in Picca et al. (2011, 2012), is also included in the extended formulation of HTPK.

The paper is organized as follows. In Section 2, the mathematical problem of reactor kinetics is presented and the limiting cases of HTPK (i.e., classical PK and multi-collision) are reviewed. In Section 3, the HTPK model is derived and its features discussed and in Section 4 a set of HTPK transient calculations is reported together with a comparison between HTPK, multi-collision with PK and reference transport calculations. Several considerations on the HTPK are detailed in Section 5, before drawing some conclusions in Section 6.

2. Reactor kinetics equations and classical solution methods

The time-dependent linear Boltzmann equation for mono-energetic particles can be written as follows (Akcasu et al., 1971; Hetrick, 1971):

$$\begin{cases} \frac{1}{v}\frac{\partial}{\partial t}\phi(\vec{x},\vec{\Omega},t)+\mathbf{L}(\vec{x},t)\phi(\vec{x},\vec{\Omega},t)=\left[\mathbf{M}_s(\vec{x},t)+(1-\beta_{\text{tot}})\mathbf{M}_f(\vec{x},t)\right]\phi(\vec{x},\vec{\Omega},t)+\sum_{n=1}^{N}\frac{\lambda_n}{4\pi}C_n(\vec{x},t) \\ \frac{\partial}{\partial t}C_n(\vec{x},t)=-\lambda_n C_n(\vec{x},t)+4\pi\beta_n\mathbf{M}_f(\vec{x},t)\phi(\vec{x},\vec{\Omega},t) \quad \text{with } n=1,\ldots,N \end{cases} \tag{1}$$

where $\beta_{\text{tot}}=\sum_{n=1}^{N}\beta n$ and, in case of isotropic scattering, the operators are defined as:

$$\begin{aligned} \mathbf{L}(\vec{x},\vec{\Omega},t)&=\vec{\Omega}\cdot\nabla+\Sigma(\vec{x},t) \\ \mathbf{M}_s(\vec{x},t)&=\frac{\Sigma_s(\vec{x},t)}{4\pi}\oint_{4\pi}\cdot\,\mathrm{d}\vec{\Omega}' \\ \mathbf{M}_f(\vec{x},t)&=\frac{\upsilon\Sigma_f(\vec{x},t)}{4\pi}\oint_{4\pi}\cdot\,\mathrm{d}\vec{\Omega}' \end{aligned} \tag{2}$$

In the following, the space/angular dependency of the operators is assumed without explicitly reference to $\vec{x}$ and $\vec{\Omega}$ (e.g., $\mathbf{L}(\vec{x},\vec{\Omega},t)\Rightarrow\mathbf{L}(t)$) to explicitly focus on time dependency (e.g. $\mathbf{L}(t)$ and $\mathbf{L}(0)=\mathbf{L}_0$). The mathematical model for neutrons in Eq. (1) is completed by setting appropriate boundary and initial conditions:

$$\begin{aligned} &\phi(\vec{x}_S,\vec{\Omega},t)=0, \quad \text{for } \vec{\Omega}\cdot\vec{n}_S<0 \\ &\phi(\vec{x},\vec{\Omega},0)=\phi_0(\vec{x},\vec{\Omega}) \end{aligned} \tag{3a}$$

where $\vec{x}_S$ defines the system boundaries and $\vec{n}_S$ is the outward-directed surface normal. In case of initial equilibrium of precursors with neutron density, their initial concentration is:

$$C_n(\vec{x},0)=C_{n,0}(\vec{x})=4\pi\frac{\beta_n}{\lambda_n}\mathbf{M}_{f,0}\phi_0(\vec{x},\vec{\Omega}) \tag{3b}$$

The solution of Eqs. (1) and (3) represents a numerically challenging problem because a) it needs large memory requirements (simultaneous discretization of space, angle and time variable) and b) its solution involves an iteration cycles (inner iterations) at each time step. Additionally, the time-dependent problem is known to be a stiff problem because the neutron and precursor time constants typically differ by several orders of magnitude. The PK model represents a very widely-used approximation of the time-dependent transport model for coupled evolution of neutrons and precursor concentrations. Another classical option to solve Eq. (1) is the multi-collision approach, where the integral nature of initial linear Boltzmann problem is approached by simulating the transport equation collision by collision.

In order to highlight similarities and differences between PK, multi-collision method and HTPK, the point model and the basic principles of multicollision approximations are reviewed in next sections.

2.1. Point kinetics model

The basic assumption of point kinetic approximation is the factorization of the neutron flux in a shape and an amplitude (Henry and Curlee, 1958), i.e.:

$$\phi(\vec{x},\vec{\Omega},t)\approx\psi(\vec{x},\vec{\Omega})a(t) \tag{4}$$

The determination of the shape is associated to a reference problem and an adjoint problem is used for the definition of the weight used in the projection, i.e.:

$$\begin{aligned} &\left[\mathbf{L}_0-\mathbf{M}_{s,0}-\mathbf{M}_{f,0}\right]\psi(\vec{x},\vec{\Omega})=0 \\ &\left[\mathbf{L}_0^{+}-\mathbf{M}_{s,0}^{+}-\mathbf{M}_{f,0}^{+}\right]\psi^{+}(\vec{x},\vec{\Omega})=0 \end{aligned} \tag{5}$$

When projecting the initial Eq. (1) onto the adjoint solution, the following system of ordinary differential equation is obtained:

$$\begin{cases} \langle\psi^{+},\frac{1}{v}\psi\rangle\frac{\mathrm{d}a(t)}{\mathrm{d}t}+\langle\psi_0^{+},\left[\mathbf{L}(t)-\mathbf{M}_s(t)-(1-\beta_{\text{tot}})\mathbf{M}_f(t)\right]\psi\rangle a(t)=\sum_{n=1}^{N}\frac{\lambda_n}{4\pi}\langle\psi^{+},C_n(\vec{x},t)\rangle \\ \frac{\mathrm{d}\langle\psi_0^{+},C_n(\vec{x},t)\rangle}{\mathrm{d}t}=-\lambda_n\langle\psi_0^{+},C_n(\vec{x},t)\rangle+4\pi\beta_n\langle\psi_0^{+},\mathbf{M}_f(t)\psi\rangle a(t) \text{ with } n=1,\ldots,N \end{cases} \tag{6}$$

In Eq. (5) the inner product is defined as:

$$\langle g,h\rangle=\int_V\mathrm{d}\vec{x}'\oint_{4\pi}\mathrm{d}\vec{\Omega}'g\left(\vec{x}',\vec{\Omega}'\right)h\left(\vec{x}',\vec{\Omega}'\right) \tag{7}$$