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Topographical clearing differential evolution: A new method to solve multimodal optimization problems

Wagner F. Sacco [a,*], Nélio Henderson [b], Ana Carolina Rios-Coelho [a]

[a] Instituto de Engenharia e Geociências, Universidade Federal do Oeste do Pará, Av. Vera Paz, s/n, Santarém, PA 68135-110, Brazil
[b] Depto. de Modelagem Computacional, Instituto Politécnico, Universidade do Estado do Rio de Janeiro, R. Bonfim, 25, Nova Friburgo, RJ 28625-570, Brazil

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ABSTRACT

Some optimization problems in the field of nuclear engineering, as for example incore nuclear fuel management and a nuclear reactor core design, are highly multimodal, requiring techniques that overcome local optima, which can be done using niching methods. In order to do so, we present a new niching method based on the clearing paradigm, Topographical Clearing, which employs a topographical heuristic introduced in the early nineties, as part of a global optimization method. This niching method is applied to differential evolution, but it can be used in other evolutionary or swarm-based methods, such as the genetic algorithm and particle swarm optimization. The new algorithm, called TopoClearing-DE, is favorably compared against the canonical version of differential evolution in two test problems: the aforementioned core design and the turbine balancing problem, which is an NP-hard combinatorial optimization problem that can be used to assess the potential of an algorithm to be applied to fuel management optimization. As the problems attacked are quite challenging, the results show that Topographical Clearing can be applied to populational optimization methods in order to solve nuclear science and engineering problems.

1. Introduction

Some optimization problems in the field of nuclear engineering are highly multimodal, remaining a great challenge for most methods. The most notorious problem is the incore fuel management (Carter, 1997; Turinsky, 2010), which is a large search space problem with $\sim 10^{12}$ possible configurations and $\sim 10^{10}$ local optima (Galperin, 1995).

Another multimodal problem is a nuclear reactor core design optimization introduced by Pereira et al. (1999), which has been attacked by other researchers (Sacco et al., 2004; Domingos et al., 2006; for example). In this work, we address the latter problem, and also an NP-hard (Garey and Johnson, 1979) problem that belongs, as well as nuclear fuel management, to the class of combinatorial optimization problems (Papadimitriou and Steiglitz, 1998): the turbine balancing problem (Mosevich, 1986). Therefore, optimization algorithms that are successful in this problem are prone to perform well in the nuclear problem.

In these multimodal problems, the search space should be thoroughly explored so that the optimization algorithm does not converge to a local optimum. To overcome this difficulty, many solutions have been proposed: a parallel genetic algorithm (Pereira and Lapa, 2003), a niching method (Mahfoud, 1995) applied to genetic algorithms (Sacco et al., 2004), a hybrid algorithm that alternates exploration and exploitation of the search space (Sacco et al., 2008), and a new mutation scheme (Sacco and Henderson, 2014) applied to differential evolution (Storn and Price, 1997).

Niching methods are techniques designed to maintain populational diversity in evolutionary or swarm-based methods, so that multiple optima are determined in multimodal problems. These optima may consist in more than one global optimum and some local minima, or in a single global optimum and many local minima. Most niching methods are based on one of the following schemes:

1. Fitness sharing (Goldberg and Richardson, 1987), which modifies the search landscape by reducing the payoff in densely populated regions (Sareni and Krähenbühl, 1998).
2. Crowding (De Jong, 1975), where a new individual replaces its most similar element in the population.
3. Clearing (Pétrowski, 1996), where the best members of the population, the so-called dominants, receive the entire payoff.

* Corresponding author. Instituto de Engenharia e Geociências, Universidade Federal do Oeste do Pará, Rua Vera Paz, s/n, Salé, Santarém, PA 68035-110, Brazil. Tel.: +55 93 8402 6503; fax: +55 93 2101 4902.
E-mail addresses: wagner.sacco@ufopa.edu.br, wagner.sacco@pq.cnpq.br (W. F. Sacco).

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The three main niching methods have been applied to the differential evolution algorithm, which we use in this work. See, for example, Thomsen (2004), Yang et al. (2008), and Qu et al. (2012). For a brief survey, see Das and Suganthan (2011). For a more detailed exposition, the reader should refer to Rönkkönen (2009).

Sareni and Krähenbühl (1998) tested these three niching schemes applied to the genetic algorithm, concluding that clearing is the best, provided that the niching radius σ that delimits each dominant's territory is correctly estimated. This is the drawback of this method, especially in real-world problems, where the search space is generally unknown beforehand.

In order to overcome this limitation, Sacco et al. (2004) proposed a variant of clearing where the individuals are clustered using Fuzzy Clustering Means (FCM, Bezdek, 1981) and each cluster has a dominant individual. However, FCM requires the number of clusters as input and is rather complicated.

With the same motivation, Qu et al. (2012) proposed an ensemble of clearing differential evolution algorithms, where the initial population is divided into three equal subpopulations P_1, P_2, and P_3, which receive radii $\sigma_{P_1} = 0.005$*SR, $\sigma_{P_2} = 0.01$*SR, and $\sigma_{P_3} = 0.05$*SR, where SR is the problem's search range. These subpopulations exchange information during the selection phase. This scheme increases clearing's efficiency, but is still dependent of σ.

In this paper, we propose a method based on the clearing paradigm which is simpler than the schemes introduced by Sacco et al. (2004) and Qu et al. (2012). It uses a clustering heuristic based on the topographical information on the objective function, which was part of an optimization algorithm proposed by Törn and Viitanen (1992), the Topographical Algorithm (TA). Recently, Sacco and Henderson (2014) used this heuristic in a new mutation operator applied to DE. In this work, we employ the topographical heuristic with the purpose of determining the dominant individual in a neighborhood. Originally, Törn and Viitanen (1992) used this mechanism to determine minima from a set of sampled points, so that they were initial solutions for a local optimization algorithm. We apply this clearing variant, called topographical clearing, to differential evolution, which outperformed the more popular genetic algorithm and particle swarm optimization in extensive experiments (Vesterstrøm and Thomsen, 2004). However, this method can be applied to any evolutionary or swarm-based technique.

The remainder of the paper is described as follows. The optimization problems are described in Section 2. The description of DE is presented in Section 3. The new niching method is introduced in Section 4, as well as its application to DE. The computational experiments and their discussions are in Section 5. Finally, the conclusions are made in Section 6.

2. The optimization problems

2.1. The nuclear reactor core design problem

Let us describe the optimization problem (for a more detailed exposition, see Pereira et al., 1999): consider a cylindrical 3-enrichment-zone reference reactor, with a typical cell composed by moderator (light water), cladding and fuel. Fig. 1 illustrates such reactor. The design parameters that may be varied in the optimization process, as well as their variation ranges, are shown in Table 1. The materials are represented by discrete variables.

The objective of the optimization problem is to minimize the average flux or power peaking factor, f_p, of the proposed reactor, allowing the reactor to be sub-critical or super critical ($k_{eff} = 1.0 \pm 1\%$), for a given average flux ϕ_0. Let $\mathbf{X} = \{R_f, \Delta_c, R_e, E_1, E_2, E_3, M_f, M_c\}$ be the vector of design variables. Then, the optimization problem may be written as

Minimize

$f_p\,(\mathbf{X})$

Subject to:

$$\phi(\mathbf{X}) = \phi_0; \tag{1}$$

$$0.99 = k_{eff}(\mathbf{X}) = 1.01; \tag{2}$$

$$\frac{dk_{eff}}{dV_m} > 0; \tag{3}$$

$$X_i^l \le X_i \le X_i^u, 1, 2\ldots, 6 \tag{4}$$

$$\mathrm{Mf} = \{\mathrm{UO_2\ or\ U-metal}\}; \tag{5}$$

$$\mathrm{Mc} = \{\mathrm{Zircaloy-2,\ Aluminium\ or\ Stainless\ Steel-304}\}, \tag{6}$$

where V_m is the moderator volume, and the superscripts l and u indicate respectively the lower and upper bounds (of the feasible range) for each design variable.

The HAMMER system (Suich and Honeck, 1967) was used for cell and diffusion equations calculations. It performs a multigroup calculation of the thermal and epithermal flux distribution from the integral transport theory in a unit cell of the lattice (Duderstadt and Hamilton, 1976),

$$\phi(\mathbf{r}) = \int_V \frac{e^{-\Sigma_t|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|^2}\cdot S(\mathbf{r}')\cdot d^3\mathbf{r}'. \tag{7}$$

The integral transport equation for scalar flux $\varphi(\mathbf{r})$, where $\mathbf{r}$ is the position vector, is solved for all sub-regions of the unit cell, being the neutron source $S(\mathbf{r})$ isotropic into the energy group under consideration. The transfer kernel in Equation (7) is related to the collision probabilities for a flat isotropic source in the initial region. The solution is initially performed for a unit cell in an infinite lattice. The integral transport calculation is followed by a multigroup Fourier transfer leakage spectrum theory in order to include the leakage effects in the previous calculation and to proceed with the multigroup flux-volume weighting.

Using the four group constants obtained from the mentioned procedure, a one-dimensional multi-region reactor calculation is performed. The diffusion equation (Duderstadt and Hamilton, 1976) is, then, solved to perform standard criticality calculation,

$$-\vec{\nabla} D_g(\mathbf{r})\vec{\nabla}\phi_g(\mathbf{r}) + \Sigma_{t,g}(\mathbf{r})\phi_g(\mathbf{r}) = \sum_{g'=1}^{4}\left[\frac{1}{k_{eff}}\chi_{g'}\Sigma_{fg'}(\mathbf{r}) + \Sigma_{sg'g}(\mathbf{r})\right]\phi_{g'}(\mathbf{r}), \tag{8}$$

where $\mathbf{r}$ is the position vector; D_g is the diffusion coefficient for group g; φ_g is the neutron flux for group g; $\sum_{t,g}$ is the total group cross section for group g; k_{eff} is the effective multiplication factor; $\chi_{g'}$ is the group fission spectrum for group g'; $\sum_{fg'}$ is the fission group cross section for group g'; $\sum_{sg'g}$ is the scattering cross section form group g' to g, and $\phi_{g'}$ is the neutron flux for group g'.

The flux $\varphi_g(\mathbf{r})$ is calculated assuming normalized source density. Equation (8) is solved using the finite difference method and a computational mesh with constant spacing in the spatial coordinate.