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Effect of void fraction covariance on relative velocity in gas-dispersed two-phase flow

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ABSTRACT

In the two-fluid model the dependence between the phases is given in the field equations by interaction terms which become a key focus for improving the overall model performance. Of the interfacial terms in the one-dimensional two-fluid model, the most important is the constitutive relation for the interfacial momentum transfer, specifically the steady-state drag force. The one-dimensional steady-state drag force is a function of area-averaged local relative velocity. This area-averaged local relative velocity can be derived from the drift-flux general expression and the relation between drift velocity and relative velocity. However, due to area averaging there is a void fraction covariance which current and past researchers have assumed to be one. Similarly, in the one-dimensional modified two-fluid model which divides the gas phase into two-groups (i.e. spherical/distorted bubbles as group-1, and cap/slug/churn-turbulent bubbles as group-2) the group-1 and group-2 area-averaged local relative velocity is required for the group-1 and group-2 steady-state drag force. These relative velocities introduce three covariance terms: group-1 void fraction, group-2 void fraction, and an inter-group covariance between group-1 and group-2 void fraction. The covariance terms have been analyzed with a substantial database from the literature including upward flow in pipe diameters of 1.27 cm–15.2 cm, downward flow in pipe diameters of 2.54 cm and 5.08 cm, and upward flow in a 1.90 cm hydraulic diameter annulus channel. Simple relations are proposed to specify the covariance in order to improve the prediction of area-averaged local relative velocity in the classical two-fluid model and the modified two-fluid model. These relations are shown to have good agreement with the experimental data in predicting the effect on the area-average relative velocity with an average relative error of 5% over the data range.

1. Introduction

By considering separate momentum and energy equations for each phase, the two-fluid model is capable of accounting for the dynamic and non-equilibrium interactions between phases in two-phase flow. While this introduces a high degree of complexity compared to the drift-flux model, the two velocity fields allow accurate two-phase modeling even when the phases are weakly coupled and therefore a valuable tool for many complex engineering systems. In the two-fluid model the dependence between the phases is given in the field equations as interaction terms which become a key focus to improving the overall model performance (Ishii, 1975; Ishii and Hibiki, 2010). For many applications, a three-dimensional understanding of the flow field is unnecessary and an area-averaged approach is adopted. Of the interfacial terms in the one-dimensional two-fluid model, the most important is the constitutive relation for the interfacial shear, specifically the steady-state drag force component (Ishii and Chawla, 1979; Hibiki and Ishii, 2010).

The one-dimensional steady-state drag force is a function of area-averaged local relative velocity. The field equations of the two-fluid model give the void weighted phase velocities which alone do not describe the area-averaged local relative velocity. Therefore, Ishii and Mishima (1984) derive the proper area-average relative velocity as a function of the void-weighted velocities and void fraction distribution parameter using the drift-flux general expression by Zuber and Findlay (1965). The area-average local relative velocity expression compensates for the slip due to the phase and velocity distribution which in most cases is much greater than the local slip. Without this consideration, the performance of the one-dimensional two-fluid model is drastically affected in the dispersed flow regimes (Ishii and Mishima, 1984). However, this

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Nomenclature

C_i	drag coefficient
C_α	covariance in void fraction
$C_\alpha{}'$	void fraction covariance factor of area-averaged relative velocity
C_0	distribution parameter
$C_{\infty,\alpha}$	asymptotic value of the void fraction covariance
$C_{\infty,0}$	asymptotic value of the distribution parameter
$D_{\mathrm{d,max}}$	maximum distorted bubble size
D_{h}	channel hydraulic diameter
G	gap characteristic length
g	gravity constant
j	volumetric flux
$M_{\mathrm{i}}^{\mathrm{D}}$	interfacial drag force
m_α	void fraction power-law exponent
m_j	volumetric flux power-law exponent
P	pressure
R^2	statistical *R*-squared
R_{o}	channel outer radius
R_{i}	channel inner radius
R_{p}	profile radius
r	radial coordinate
v_{gj}	drift velocity
v	velocity
v_{r}	local relative velocity
V_{r}	relative velocity
x_{p}	profile radius ratio

Greek symbols

α	void fraction
α_0	center line void fraction
Δ	difference
ρ	density
σ	surface tension

Subscripts

1	group-1 property
2	group-2 property
g	gas phase
f	liquid phase
k	*k*-phase
crit	critical
cor	correlation value
exp	experimental value
n	*n*th bubble group

Mathematical symbols

$\langle\rangle$	area averaged quantity
$\langle\langle\rangle\rangle$	void fraction weighted area averaged quantity
$\vert\ \vert$	absolute value

formulation results in a void fraction covariance dependence due to the area averaging which represents the ratio between the mean of the product of void fraction and product of the mean void fraction. This term has not been adequately addressed in literature and as a result has been widely neglected by assuming this covariance is one. This is also true of computational system analysis codes such as TRACE (U.S. NRC, 2008) and RELAP5/MOD3 (RELAP5/MOD3.3, 2001) which utilize the drift-flux approach for interfacial drag, as well as, TRAC-PF/MOD1 (Liles et al., 1988) which uses the Ishii and Mishima (1984) relative velocity formulation in the drag coefficient approach for interfacial drag (Brooks et al., 2012a).

The interaction terms in the two-fluid model are also a strong function of interfacial area concentration which led to the development of the interfacial area transport equation (IATE) in order to correctly describe the dynamic changes in interfacial area concentration (Kocamustafaogullari and Ishii, 1995). A two-bubble-group approach to IATE (Hibiki and Ishii, 2000) was soon considered due to the bubble size and shape effects on the transport phenomena, specifically the bubble drag and interaction mechanisms (Sun et al., 2003). The two-fluid model with two-group IATE extends the number of gas fields to two where spherical/distorted bubbles are modeled together as group-1 and cap/slug/churn-turbulent bubbles as group-2, requiring separate mass, momentum and energy equations for each group. In the one-dimensional formulation, the momentum equations for both bubble groups require constitutive relations for steady-state drag force and therefore separate equations defining the area-averaged relative velocity for each bubble group. The two-group formulation of relative velocities was addressed by Brooks et al. (2012b) by using a similar approach as Ishii and Mishima (1984). As to be expected, the two-group approach introduces void covariances from both bubble groups, as well as, an inter-group void fraction covariance.

In view of the lack of understanding of the effects from void fraction covariance on relative velocity, the several covariances mentioned are studied with experimental data. The objective of this study is to analyze the effect of the covariance in void fraction in order to improve the overall prediction of the two-fluid model within both the traditional and two-group formulation through easily implemented modifications to the existing closure relations.

2. Existing work

2.1. One-group area-averaged relative velocity

Ishii and Mishima (1984) showed that the area-averaged relative velocity can be derived from the void weighted drift velocity using the one-dimensional drift-flux model. The void weighted drift velocity represents the local slip between phases and is given in terms of relative velocity by

$$\langle\langle v_{gj}\rangle\rangle \equiv \frac{\langle \alpha v_{gj}\rangle}{\langle\alpha\rangle} = \frac{\langle \alpha(1-\alpha)v_{\mathrm{r}}\rangle}{\langle\alpha\rangle} \tag{1}$$

where v_{gj} is the drift velocity, α is void fraction and v_{r} is local relative velocity. Here, $\langle\langle\rangle\rangle$ are used to denote area-averaged void weighted term and $\langle\rangle$ identifies a simple area-average. The void weighted drift velocity is also given by the drift-flux model (Zuber and Findlay, 1965) as

$$\langle\langle v_{gj}\rangle\rangle = \langle\langle v_{\mathrm{g}}\rangle\rangle - C_0\langle j\rangle \tag{2}$$

where v_{g}, C_0, j are the gas velocity, distribution parameter and volumetric flux respectively. The second term of Eq. (2) represents the effect of void fraction and volumetric flux distributions across the flow area. If the relative velocity is assumed to be uniform across the flow area, the void weighted drift velocity can be written as

$$\langle\langle v_{gj}\rangle\rangle \simeq \frac{\langle\alpha(1-\alpha)\rangle}{\langle\alpha\rangle}\langle v_{\mathrm{r}}\rangle = \left(1-\frac{\langle\alpha^2\rangle}{\langle\alpha\rangle}\right)\langle v_{\mathrm{r}}\rangle. \tag{3}$$