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Comments on the application of bifurcation analysis in BWR stability analysis

Carsten Lange*, Dieter Hennig, Manuel Schulze, Antonio Hurtado

Department of Hydrogen and Nuclear Energy, Institute of Power Engineering, Faculty of Mechanical Engineering, Technische Universität Dresden, George-Bähr-Str. 3b, 01069 Dresden, Germany

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ABSTRACT

Currently, BWR stability analysis is most often performed by the application of system codes which provide the time evolution of the neutron flux or thermal power at a defined operational point (OP) after imposing a system parameter perturbation. However, in general it is impossible to understand the real stability state of the BWR at a specific OP by the application of system code analysis alone. Hence, we are exploring methods developed in the nonlinear dynamics field in order to reveal the nature of the BWR stability states when power oscillations are observed. A powerful method is bifurcation analysis. In order to motivate this "nonlinear thinking" versus "linear thinking", in this paper we will demonstrate some examples of phenomena which can only be understood in nonlinear terms by application of bifurcation theory and where linear interpretation leads to incorrect conclusions.

1. Introduction

In Boiling Water Reactor (BWR) stability analysis it is common practice to measure or calculate the so-called Decay Ratio (DR) at the intrinsic (or natural) frequency, NF, of the BWR as a reliably measurable technical stability indicator. The DR, strictly speaking the asymptotic DR[1] (Hennig, 1999), is a reliable linear stability indicator characterising the oscillatory stability behaviour of a linear (or linearised) dynamical system. Furthermore, in the theory of nonlinear dynamical systems, the theorem of Hartman and Grobman (Guggenheimer and Holmes, 1984) allows the application of linear stability indicators to determine the stability properties of hyperbolic fixed points (operational points, equilibrium points) of nonlinear dynamical systems. In this case there are, beyond DR estimation, many powerful methods to examine the system stability state, such as Nyquist plots, root locus criterion and others (Hetrick, 1971).

Due to the nonlinearity of BWR dynamics we are interested in the nonlinear stability behaviour. The term stability analysis comprises the examination of the stability of reactor states as equilibrium points (fixed points) and periodic orbits (limit cycles), or states where equilibrium points and periodic orbits coexist. The application of the bifurcation theory could be very helpful in the stability analysis of nonlinear dynamical systems. This approach has been applied in BWR reactor dynamics since the mid 1980s (van Bragt, 1998; Tsuji et al., 1993; Uddin, 1981; Munoz-Cobo and Verdu, 1991). Some of the authors combined bifurcation codes with simplified BWR models, so-called Reduced Order Models (ROMs). The first systematic bifurcation analysis was published by Munoz-Cobo and Verdu (1991). They used a very simple BWR model published in March-Leuba et al. (1986), but the analytical approach in Munoz-Cobo and Verdu (1991) already demonstrates the essence of the bifurcation analysis of dynamical systems with dimensions larger than 2 (e.g. Hopf bifurcations, centre manifold reduction, transformation to Poincare normal form). In (van Bragt, 1998; Uddin, 1981; Zhou and Uddin, 2002; Dokhane, 2004; Lange, 2009), advanced ROMs were coupled with the bifurcation code BIFDD developed by Hassard et al. (1981). However, semi-analytical nonlinear BWR stability analysis without reference to bifurcation analysis were also published (March-Leuba et al., 1986; Akcasu et al., 1971).[2,3]

* Corresponding author. Tel.: +49 351 463 32083; fax: +49 351 463 37161.
E-mail addresses: carsten.lange@tu-dresden.de, carstencumulus@gmx.de (C. Lange).

[1] The asymptotic DR is calculated from the poles of the transfer function of the linearised system equations lying nearest to the unit circle (Hennig, 1999).

[2] Note that many studies of nonlinear stability analysis of 2-phase flow with and without reference to bifurcation analysis were published in the 1970s/80 s, e.g. (Karve, 1998; Podowski, 1992; Achard et al., 1985).

[3] Obviously, all system code stability analyses conducted by time domain codes solving partial nonlinear differential equations are nonlinear analyses.

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In the last few years an advanced ROM was developed at PSI and TUD (Dokhane, 2004; Lange, 2009), which is partially based (the thermal-hydraulics channel model and the fuel dynamics) on the ROM previously developed at University of Illinois (Uddin, 1981; Karve et al., 1997). This ROM was qualified for practical BWR stability analysis (Lange, 2009; Lange et al., 2011) by model improvements and input data adjustments (to system code steady-state results). Hence it was meaningful to couple this ROM with a bifurcation analysis code (in this case BIFDD (Hassard et al., 1981))[4] and apply the ROM-BIFDD and a validated system code (like RAMONA 5-2 (Wulff et al., 1984)) side by side in a complementary sense. We called this procedure the ROM-RAM method (Lange et al., 2011). The method provides, in a first (ROM) step, an overview of the stability landscape in the vicinity of the selected BWR operational point and uses, in a second step (RAM), a system code (RAM) containing a sufficiently detailed BWR model for more precise calculation. Hence, from the first step we know what we have to expect, and the unambiguous interpretation of the system code results should be easier. Taking into account that the BWR stability analysis is a nonlinear dynamics problem and that the linear analysis is a specific subset of it (theorem of Hartman–Grobman (Guggenheimer and Holmes, 1984)), the RAM-ROM methodology is, from our point of view, an appropriate approach for a comprehensive understanding of the dynamical system stability.

In order to motivate this “nonlinear thinking” versus “linear thinking”, in this paper we will demonstrate some examples of phenomena which can only be understood in nonlinear terms, and where linear interpretation leads to incorrect conclusions. First, we will demonstrate that the stability state will change discontinuously if a nonlinear dynamical system under parameter variation encounters a Hopf bifurcation. This discontinuity in the stability behaviour even occurs though linear stability indicators like DR may formally change continuously (the DR will lose its property as a stability indicator if a non-hyperbolic fixed point is reached). If in a BWR core, e.g. two spatial neutron flux modes (power modes) are excited to oscillate and both modes are differently stable, a DR measurement by the monitoring system will return a jump in the DR history if the measurement device sensitivity is high enough with respect to the limit cycle mode (limit cycle amplitude is dominant). This DR jump is not explainable by a linear model (Pàzsit, 1995) because in a linear system a limit cycle oscillation state does not exist. Thus, the explanation of the DR jump discussed e.g. in Pàzsit (1995) is correct for cases where both spatial modes are stable but oscillate with different magnitudes, and the DR's are significantly different (hyperbolic fixed points). Furthermore, we will show that a jump in the DR can be observed when the BWR is working in a post-subcritical bifurcation regime characterised by the coexistence of different stability states (stable fixed point, stable and unstable limit cycles). In this scenario, a turning-point must exist. By reaching this turning point, the DR may jump to unity under parameter variation if the monitoring system first returns the DR with respect to the stable fixed point (for not too large parameter perturbations, see Section 5).

In order to demonstrate that “thinking in the linear world” can lead to misinterpretation, we add a non-nuclear example (appendix C). The Tacoma bridge failure in the 1930s in the USA is a typical instability event which was at first misinterpreted as a linear resonance problem. Detailed investigations (Parkinson and Smith, 1964; Novak, 1971; Thomson, 1982) revealed the correct reasons of this failure as a complex stability multi-state, a saddle node bifurcation of cycles.

[4] In the meantime there are more powerful bifurcation analysis codes available.

2. Remarks to linear and nonlinear stability analysis

Generally, stability analysis is the investigation of the temporal behaviour of state variables after an internal or external perturbation is imposed on the dynamical system, while one or more system parameters will be varied in their domain of definition. If the system is stable, all state variables converge to the equilibrium point (singular fixed point or in its close neighbourhood, also called “Lyapunov stability” (Hetrick, 1971; Akcasu et al., 1971)). If the system is unstable, at least one of the state variables is diverging in an oscillatory or exponential manner. The critical value of the system parameter(s) which separate stable fixed points from unstable ones is the so-called stability boundary. In the following (see also Appendix A) we summarise some definitions and results of the bifurcation theory for the readers convenience whereupon the phenomena Hopf bifurcation and generalised Hopf bifurcation are of greatest importance for the understanding of our concern.

2.1. Dynamical system, state space, orbit, fixed point, limit cycle

For a mathematical description we follow (Guggenheimer and Holmes, 1984; Hetrick, 1971; Kuznetsov, 1998; Seydel, 2012; Neyfeh and Balachandran, 1995; Strogatz, 1994). A BWR loop constitutes a nonlinear dynamical system described by a set of partial differential equations

$$\frac{\partial \vec{X}(t)}{\partial t} = \vec{F}\left(\partial^n \vec{X}(t), \ldots, \vec{X}(t), \vec{\gamma}\right) \tag{1}$$

in the case of distributed parameter systems like system codes[5] or ordinary differential equations

$$\frac{\partial \vec{X}(t)}{\partial t} = \vec{F}(\vec{X}(t), \vec{\gamma}) \tag{2}$$

in the case of lumped parameter systems (reduced order model equation sets[6]). $\vec{X}(t)$ with $\vec{X}\in \mathbb{R}^n$ is a state vector, $\vec{F}$ (with $\vec{F}:\mathbb{R}^n\times\mathbb{R}\to\mathbb{R}^n$ is C^∞) is a vector field, $\vec{\gamma}\in \mathbb{R}^m$ is a parameter vector (also called control parameter vector with m components) and $t\in\mathbb{R}$ is the time. The number of all possible system states of (2) is called the state space $X\in\mathbb{R}^n$.

In the following, we analyse the stability of an autonomous ($\vec{F}$ is not explicitly time-dependent) time-continuous dynamical system (2). For this purpose, we consider the evolution that determines the system state $\vec{X}(t)$ at time t if the initial state $\vec{X}_0 = \vec{X}(0)$ is known. The curve in $\mathbb{R}^n$ connecting both states is called the trajectory or orbit of $\vec{X}_0$ and the association $\vec{X}(0)\to\vec{X}(t)$ defines the flow $\Phi^t:\mathbb{R}^n\to\mathbb{R}^n$, where $\Phi^t(\vec{X}(0))\equiv\Phi(\vec{X}(0),t) = \vec{X}(t)$. With the definition of the flow, we can define fixed points (steady state solution) and limit cycles (periodic solution). The point $\vec{X}_0$ is called the fixed point, if $\Phi^t(\vec{X}_0) = \vec{X}_0$ for all t. In this case, the steady state solution $\vec{X}_0$ satisfies the equation

$$\frac{d\vec{X}_0}{dt} = 0 = \vec{F}(\vec{X}_0, \vec{\gamma}). \tag{3}$$

In contrast to a fixed point, a cycle is a non-equilibrium (periodic) orbit C, such that each point $\vec{X}_{c}\in C$ satisfies

[5] System code: integrated BWR loop model, the coupled neutron kinetic and thermal-hydraulic PDE's are solved.

[6] Reduced order model: spatial averaged PDE but the stability properties of the resulting ordinary DE (ODE) are very similar to the PDE of the system code equations.