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# Composite spatial grid spectral nodal method for one-speed discrete ordinates deep penetration problems in *X*,*Y* geometry

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A B S T R A C T

Computer modeling of radiation deep penetration problems is historically based on the discrete ordinates ($S_N$) formulation. For efficiency reasons, besides accuracy, coarse-mesh spatial discretization is desirable. The spectral Green's function (SGF) methods form a class of accurate coarse-mesh numerical methods as they use polynomial approximations only for the node-edge transverse leakage terms; the scattering source terms are treated analytically in the numerical algorithm. Therefore, algebraic work and the computational algorithms of the spectral nodal methods are rather complicated. To alleviate this negative feature, we offer in this paper a composite spatial grid SGF nodal method for the numerical solution of one-speed deep penetration $S_N$ problems with isotropic scattering in *X*,*Y* geometry. This method uses a rectangular coarse spatial grid, that is coincident with the material region distribution within the shielding structure. We first transverse integrate the $S_N$ equations separately in the *x*- and *y*-coordinate directions inside each material region, and then we introduce flat approximations for the transverse leakage terms. Furthermore, we use a fine spatial grid to discretize each set of "one-dimensional" $S_N$ nodal equations. As the spatial directions are coupled by the transverse leakage terms, we use an explicit alternate direction technique to converge the numerical solution. In order to verify the offered method's accuracy, we present numerical results for typical model problems. Moreover, we compare the computing performance of this method with the conventional SGF-constant nodal method.



## 1. Introduction

Deep penetration problems appear in several nuclear science and engineering phenomena. Basically they are problems where particle interaction with target nuclei of the medium does not multiply the number of particles. The main application is shielding calculations for radiation protection; however, other applications, such as nuclear medicine, material science and geophysical studies have evolved over the years.

In this context, the discrete ordinates ($S_N$) formulation of the monoenergetic neutron transport equation is the conventional mathematical model for deterministic computational simulation of deep penetration problems (Lewis and Miller, 1993). The first step of the $S_N$ numerical schemes is the spatial discretization. For efficiency reasons, besides accuracy, coarse-mesh discretization is also desirable. Nodal methods offer accurate results in coarse-mesh calculations. In these methods, the $S_N$ equations are transverse integrated and analytically solved by introducing polynomial approximations for the node-edge transverse leakage and scattering source terms. On the other hand, the spectral Green's function (SGF) methods form a class of accurate spectral nodal methods as they use polynomial approximations only for the node-edge transverse leakage terms. The source terms are treated analytically in the numerical algorithm. Consequently, spectral nodal methods generate more accurate results than the conventional nodal methods (Barros and Larsen, 1992; Dominguez and Barros, 2007). On the other hand, the algebraic work and the computational algorithms of the spectral nodal methods are rather complicated. To alleviate this negative feature, we present in this work a composite spatial grid spectral Green's function (CSG-SGF) nodal method for the numerical solution of one-speed discrete ordinates ($S_N$) deep penetration problems with isotropic scattering in *X*,*Y* geometry.

In the CSG-SGF method, the spatial variable discretization scheme combines a non-conventional composite grid method with the traditional transverse integration procedure of the $S_N$ nodal methods (Badruzzaman, 1990). We use a rectangular coarse spatial grid that is coincident with the material region distribution and within this grid we transverse integrate the $S_N$ equations. Then, we introduce constant approximations for the transverse leakage terms yielding two "one-

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dimensional" problems, coupled by these leakage terms. In addition, we use a fine spatial grid to discretize each "one-dimensional" $S_N$ nodal equation, using the one-dimensional SGF method (Barros and Larsen, 1990), that treats analytically the scattering source terms and generates numerical solutions that are completely free of spatial truncation errors. As the spatial directions are coupled by the transverse leakage terms, we use an explicit alternating direction technique to converge the numerical solution.

In the next section we present the mathematical basis on the present CSG-SGF nodal method with constant approximation for the transverse leakage terms (CSG-SGF-CN). In Section 3, we describe the numerical iterative scheme used in the CSG-SGF-CN method. In Section 4, we show the numerical results for typical deep penetration problems. We conclude, in Section 5 with a discussion and suggestions for future work.

## 2. Mathematical basis of the CSG-SGF-CN method

Let us consider a rectangular domain with several regions $R_{i,j}$ of height $L_{y,j}$ and width $L_{x,i}$ as illustrated in Fig. 1. Each region $R_{i,j}$ has constant material properties and the $S_N$ equations within region $R_{i,j}$ are

$$\mu_m\frac{\partial\psi_m(x,y)}{\partial x}+\eta_m\frac{\partial\psi_m(x,y)}{\partial y}+\sigma_{t_{i,j}}\psi_m(x,y) = \frac{\sigma_{s_{i,j}}}{4}\sum_{n=1}^{M}\psi_n(x,y)\omega_n+Q_{m,i,j} \tag{1}$$

for $(x,y)\in R_{i,j}$ with $i=1,\ldots,I$ and $j=1,\ldots,J$. In Eq. (1), $m=1,\ldots,M$, $M=N(N+2)/2$; $\mu_m$ and $\eta_m$ are the $S_N$ angular directions, $\omega_n$ are the angular quadrature weights, $\sigma_t$ represents the total macroscopic cross section, $\psi_m$ is the neutron angular flux in the $(\mu_m,\eta_m)$ direction, $\sigma_s$ is the isotropic scattering macroscopic cross section and $Q_m$ is a uniform interior source of neutrons in direction $m$.

At this point, we describe the first discretization scheme, introducing an outer coarse spatial grid $\Lambda$ whose nodes are coincident with the material regions, meaning that any material region $R_{i,j}$ represent a spatial coarse discretization cell $\Lambda_{i,j}$ in the outer coarse grid showed Fig. 1. To obtain the transverse-integrated $S_N$ equations on the coarse grid $\Lambda$, we apply the operator

$$\frac{1}{L_{y,j}}\int_{y_{j-1/2}}^{y_{j+1/2}}\cdot dy \tag{2}$$

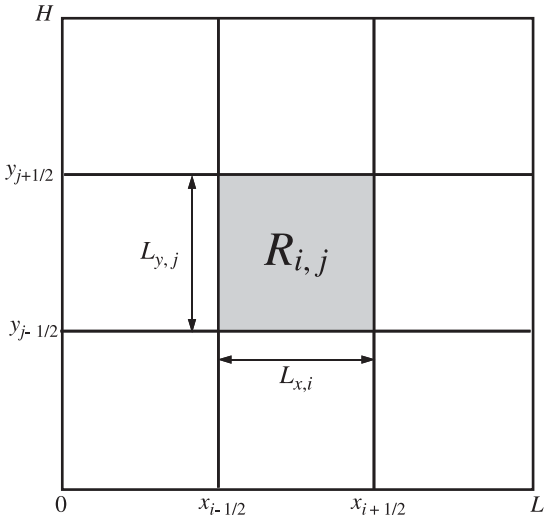


**Fig. 1.** Domain of solution and outer coarse grid $\Lambda$.

to Eq. (1). The result is the $x$-direction transverse integrated $S_N$ equation

$$\mu_m\frac{d}{dx}\tilde{\psi}_{m,j}(x)+\sigma_{t_{i,j}}\tilde{\psi}_{m,j}(x) = \frac{\sigma_{s_{i,j}}}{4}\sum_{n=1}^{M}\tilde{\psi}_{n,j}(x)\omega_n+Q_{m,i,j} - \frac{\eta_m}{L_{y,j}}\left[\psi_m\left(x,y_{j+1/2}\right) - \psi_m\left(x,y_{j-1/2}\right)\right] \tag{3}$$

where we have defined

$$\tilde{\psi}_{m,j}(x)=\frac{1}{L_{y,j}}\int_{y_{j-1/2}}^{y_{j+1/2}}\psi_m(x,y)dy,\quad m=1,\ldots,M,\quad (x,y)\in R_{i,j}. \tag{4}$$

Following a similar procedure, we can obtain the $y$-direction transverse integrated $S_N$ equation. The coarse-grid transverse integrated $S_N$ equations are composed of a system of $2M$ ordinary differential equations in $4M$ unknowns, i.e., $\tilde{\psi}_{m,j}(x)$, $\widehat{\psi}_{m,i}(y)$, $\psi_m(x_{i\pm1/2},y)$ and $\psi_m(x,y_{j\pm1/2})$. Therefore, to solve this system with a unique solution, we introduce constant approximations (Barros and Larsen, 1992) for the transverse leakage terms along the region edges. The result appears as

$$\mu_m\frac{d}{dx}\tilde{\psi}_{m,j}(x)+\sigma_{t_{i,j}}\tilde{\psi}_{m,j}(x)=\frac{\sigma_{s_{i,j}}}{4}\sum_{n=1}^{M}\tilde{\psi}_{n,j}(x)\omega_n+Q_{m,i,j}-\widehat{F}^{y}_{m,i,j} \tag{5}$$

where

$$\widehat{F}^{y}_{m,i,j}=\frac{\eta_m}{L_{y,j}}\left[\widehat{\psi}_{m,i,j+1/2}-\widehat{\psi}_{m,i,j-1/2}\right],\quad m=1,\ldots,M. \tag{6}$$

Eq. (5) and the companion equation in the $y$ direction can be thought of as two one-dimensional problems whose solutions can be found separately. We obtain the analytical general solution for Eq. (5) using a spectral analysis technique as described for the conventional SGF-constant nodal (SGF-CN) method (Barros and Larsen, 1992).

To discretize these one-dimensional equations, we introduce independent fine grids $\Omega_x$ and $\Omega_y$ within regions $R_{i,j}$. The fine grid $\Omega_x$ divides each region $R_{i,j}$ into $K_i$ nodes of height $L_{y,j}$ and corresponding width $\Delta_k^{x,i,j}$. The other fine grid $\Omega_y$ divides each region $R_{i,j}$ into $P_j$ nodes of height $\Delta_p^{y,i,j}$ and corresponding width $L_{x,i}$. The geometry of the two inner fine spatial grids is represented in Fig. 2.

Moreover, we obtain the spatial balance discretized equations for the $x$-dependent problem applying the operator

$$\frac{1}{\Delta_k^{x,i,j}}\int_{x_{k-1/2}}^{x_{k+1/2}}\cdot dx \tag{7}$$

to Eq. (5). The resulting equation is

$$\frac{\mu_m}{\Delta_k^{x,i,j}}\left(\tilde{\psi}_{m,k+1/2,j}-\tilde{\psi}_{m,k-1/2,j}\right)+\sigma_{t_{i,j}}\overline{\psi}^{x}_{m,k,j} = \frac{\sigma_{s_{i,j}}}{4}\sum_{n=1}^{M}\overline{\psi}^{x}_{n,k,j}\omega_n+Q_{m,i,j}-\widehat{F}^{y}_{m,i,j} \tag{8}$$

where

$$\overline{\psi}^{x}_{m,k,j}=\frac{1}{\Delta_k^{x,i,j}}\int_{x_{k-1/2}}^{x_{k+1/2}}\tilde{\psi}_{m,j}(x)dx,\quad m=1,\ldots,M. \tag{9}$$

For each spatial cell of the inner fine grid $\Omega_x$, Eq. (8) represents a system of $M$ algebraic linear equations in $2M$ unknowns $[\tilde{\psi}_{m,k+1/2,j}$