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# Prediction of reinjection effects in fault-related subsidiary structures by using fractional derivative-based mathematical models for sustainable design of geothermal reservoirs

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ABSTRACT

This study provides a method to evaluate the effects of cold-water injection into an advection-dominated geothermal reservoir. A fractional advection-dispersion equation (fADE) and a fractional heat transfer equation (fHTE) are applied to fault-related structures in geothermal areas where the fracture density is described by a power-law model. Synthetic production data generated by a numerical reservoir simulator reveal that the fADE and the fHTE are in reasonable agreement with the tracer responses and temperature change in a fault zone. Tracer analysis based on the fADE has potential to elucidate fault-related structures and to predict premature thermal breakthroughs quickly and efficiently.



## 1. Introduction

Geothermal energy is a promising energy source for stable generation of electricity regardless of the weather or time of day. Most geothermal power plants extend the lifespan of the resources by sustaining the amount of water and pressure within the reservoir. Reinjection is of great importance for sustainable utilization of geothermal systems, which has been discussed in Axelsson (2010) and Kaya et al. (2011). One of the major problems with this reinjection process, however, is the possibility of an early thermal breakthrough in production areas. There remains a need to establish criteria and guidelines for sustainable reinjection operations that allow us to design the location of wells, injection temperatures, and/or flow rates.

Fracture and fracture networks contribute critically to fluid flow and heat propagation. In a geothermal reservoir, structures associated with large-scale faults appear to be quite important in controlling fluid flow (Massart et al., 2010). The simplest description of a fault zone structure (and fault zones in general) considers two major mechanical units, namely a fault core and a damage zone (cf., Caine et al., 1996; Faulkner et al., 2010). The fault core is formed through repeated slipping of the principal fault plane and is composed of impermeable barriers. The damage zone consists of a volume of deformed rocks with smaller fractures around a fault surface that results from slips along faults (Caine et al., 1996). In the fault damage zone, the fracture density (the number of fractures per unit length) commonly increases near the fault core (e.g., Brock and Engelder, 1977; Chester and Logan, 1986; Agosta and Kirschner, 2003; De Joussineau and Aydin, 2007; Gudmundsson, 2011). Savage and Brodsky (2011) found that isolated single faults with small displacements have macrofracture densities that decay as a power law. The power-law function is a feature of fractal geometry, which provides widely applicable and descriptive tools for characterization of subsurface fracture systems (Bonnet et al., 2001).

Tracer testing is a standard method for evaluating fluid flow within a geothermal reservoir. Tracer responses often observed in geothermal fields include non-Gaussian leading or trailing edges (also called heavy tails) of a plume emanating from a point source, or nonlinear growth of the centered second moment (e.g., Sanjuan et al., 2006). Numerous numerical experiments indicate that anomalous dispersion cannot be described by the traditional second-order advection-dispersion equation (ADE) that is based

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**Nomenclature**

| | |
|---|---|
| $a$ | arbitrary constant |
| $b$ | retardation coefficient for tracer |
| $c$ | tracer concentration |
| $C$ | non-dimensional concentration of tracer |
| $Cp$ | heat capacity |
| $d$ | arbitrary constant |
| $D$ | dispersion coefficient |
| $dx$ | grid spacing |
| $dy$ | grid spacing |
| $e$ | retardation parameter for heat |
| fADE | fractional advection-dispersion equation |
| FD | fracture density |
| fHTE | fractional heat transfer equation |
| FW | foot wall |
| $H$ | thickness of calculation domain |
| HW | hanging wall |
| $K$ | permeability |
| $l$ | representative length |
| $L$ | length of the calculation domain |
| $n$ | exponent describing decay of fracture density |
| $p$ | Skewness parameter |
| $Pe$ | Péclet number |
| $R^2$ | coefficient of determination |
| RMSE | root mean square error |
| $t$ | time |
| $t_{tracer}$ | characteristic time of tracer migration |
| $T$ | non-dimensional temperature |
| $v$ | Darcy velocity |
| $x$ | horizontal coordinate |
| $X$ | non-dimensional distance |
| $y$ | horizontal coordinate |
| $\alpha$ | order of spatial fractional derivative in fADE |
| $\beta$ | order of temporal fractional derivative in fADE |
| $\beta'$ | order of temporal fractional derivative in fHTE |
| $\gamma$ | order of temporal fractional derivative in fADE |
| $\gamma'$ | order of temporal fractional derivative in fHTE |
| $\theta$ | exponent describing decay of permeability |
| $\Theta$ | temperature |
| $\xi$ | exponent describing decay of diffusivity |
| $\mu$ | viscosity |
| $\rho$ | density |
| $\tau$ | non-dimensional time |
| $\phi$ | porosity |

*Subscripts*

| | |
|---|---|
| 0 | reference value |
| 1 | surrounding rock |
| 2 | main conduit |
| 3 | matrix |
| $i$ | node index |
| $in$ | injection |
| $o$ | initial |
| $r$ | rock |
| $w$ | water |

on Fick's diffusion law (Adams and Gelhar, 1992). To describe such tracer behaviors, several alternative mass transport models have been developed. Fractional differential equations have been demonstrated to simulate the anomalous characteristics of solute transport in highly heterogeneous media (Benson et al., 2000; Baeumer et al., 2001; Schumer et al., 2003). Using fractional derivatives in time and space, Fomin et al. (2005, 2011) derived the fractional advection-dispersion equation (fADE). The fADE accounts for the diffusion of solute into surrounding rocks based on a fractal diffusion coefficient. In the last decade, many authors have made notable contributions to both the theory and the application of fADE in hydrology, as reviewed by Zhang et al. (2009).

The transport properties of tracers cause the chemical front to be close to the thermal front. Because the chemical front of the tracers arrives earlier than the heat front does, the tracer response can be used as an indicator of thermal breakthrough. Migration of the cold-water front for a single-phase flow is fairly well understood from the work of Lauwerier (1955) and Bodvarsson (1972). Subsequently, other work has estimated temperature changes over time at the production wells (Gringarten and Sauty, 1975; Bödvarsson and Tsang, 1982; Shook, 2001; Kocabas, 2005). When advection is dominant, the retardation of the thermal front in comparison to the hydrodynamic front can be described by the ratio of rock/water volumetric heat capacities. This assumption has been widely used to estimate cooling effects of reinjected fluid in several geothermal fields but leads to overestimation of the temperature decline (e.g., Aksoy et al., 2008). There is a possibility of heat conduction into the surrounding rocks, which may play an important role in the temperature change in fractured reservoirs. In these models, the tracer is assumed to migrate along preferential flow paths (fractures), and the breakthrough time is essentially a measure of the total volume of the path between injection and production wells. Thermal migration, on the other hand, is determined by the surface area for thermal conduction from the reservoir rocks to the preferential flow path (Pruess and Bodvarsson, 1984; Shook, 2003).

López and Smith (1995) examined the interaction between thermally driven circulation in a fault zone and the surrounding rocks. They mapped fluid flow and heat transfer regimes in different permeability regions of the fault and the surrounding rocks. For fault permeabilities lower than $7 \times 10^{-13}$ m$^2$ and rock permeabilities lower than $5 \times 10^{-18}$ m$^2$, conduction was the dominant transport mechanism, while advection dominated for rock permeabilities higher than $5 \times 10^{-18}$ m$^2$. In the former case, the effect of thermal conductivity of the surrounding rocks cannot be ignored. In the latter case, the surrounding rocks are relatively permeable, which may be due to small fractures around the fault in damage zones. In this case, the thermal migration can be advective, which causes a lag behind the fluid front by a constant related to the ratio of rock/water volumetric heat capacities. Our previous study (Suzuki et al., 2014) proposed the fractional heat transfer equation (fHTE) based on the fADE to characterize diffusion into surrounding rocks by using fractional derivatives in the same manner as the fADE. The fHTE considers that thermal diffusion occurs as a result of hydrodynamic mixing of the fluid particles passing through the surrounding rocks. This heat transfer due to fluid flow is of importance when advection is dominant.

Little has been reported on quantitative investigations of fracture density and spatial distributions at a damage zone in a geothermal field. First, using data from the literature, we demonstrated that the fracture distributions in a geothermal reservoir are accurately characterized by power-law functions. Then, the relationship between the fault-related structure and the fADE and the fHTE were explained. A numerical reservoir simulator, TOUGH2, was used to generate production data in an advection-dominated reservoir to verify the applicability of the fADE and the fHTE. Finally, we proposed a method of analyzing tracer responses for characterizing fault-related structures.

## 2. Mathematical model

Field observations in geothermal reservoirs indicate that main permeable zones are typically formed by fault-related structures,