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Stochastic modelling and control of rainwater harvesting systems for irrigation during dry spells

Koichi Unami [a,*], Osama Mohawesh [b,c], Erfaneh Sharifi [a], Junichiro Takeuchi [a], Masayuki Fujihara [a]

[a] Graduate School of Agriculture, Kyoto University, Kyoto, Japan
[b] Prince Faisal Center for Dead Sea, Environmental and Energy Research, Mutah University, Karak, Jordan
[c] Department of Plant Production, Faculty of Agriculture, Mutah University, Karak, Jordan

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ABSTRACT

A generic rainwater harvesting system including a catchment area and a command area is often equipped with rainwater storage tanks. The stochastic nature of precipitation dominates water balance of rainwater harvesting systems, and the theory of stochastic control better serves for determining optimal strategies for water management. A mathematical model consisting of stochastic differential equations, with few model parameters that can be identified from observed data, is developed to describe dynamics of rainwater harvesting systems for irrigation. Stochastic control problems are formulated and then solved to obtain the optimal irrigation strategies during dry spells. This procedure can be inversely applicable to designing dimensions of a system. Identification of the model parameters is demonstrated with the data observed in an experimental micro rainwater harvesting system in Japan as well as in semi-arid savanna of Ghana. Then, a real life application is discussed in the context of the Jordan Rift Valley, where a rainwater harvesting system will be developed.

1. Introduction

Rainwater harvesting (RWH) technology is drastically improving agricultural productivity in arid and semi-arid regions, where irregular precipitation and prolonged dry spells are the major constraints (Pachpute et al., 2009). It also alleviates inadequate access to clean drinking water under conditions of poverty (De Moraes and Rocha, 2013). Mathematical methods are expected to advance development of RWH technology in the modern world. Unami et al. (2010) modeled irregularity of rainfall intensity as well as duration of dry spells using stochastic differential equations. Unami et al. (2013) discussed applicability of the stochastic control theory to establishing operational strategies for dry season irrigation with a RWH micro-dam.

A generic RWH system for irrigation purposes includes a catchment area and a command area. Rainwater storage tanks (RWSTs) such as RWH micro-dams may be installed in order to balance the demand and supply of water. Mwenge Kahinda and Taigbenu (2011) classified RWH systems as in situ RWH when the command area includes the catchment area and as ex situ RWH when the catchment area excludes the command area. The water is conveyed to and within the command area most commonly by gravity. However, under such a situation that availability of arable land is limited and the cost of lifting water is affordable, it may be a feasible option to develop a RWH system where the catchment area includes the command area.

No matter how small the scale of the RWH system is, understanding hydrological processes in the catchment area is imperative for rational management. Guerra et al. (1990) analyzed the hydrological processes in terraced rice fields with RWSTs having capacities of 1000–4000 m^3 in central Luzon, Philippines. Ngigi et al. (2005) examined rainfall-runoff relationships in the catchments of RWSTs with 30–100 m^3 capacities to evaluate the RWH system in the Laikipia district of Kenya. Panigrahi et al. (2007) experimentally studied water balance in a rainfed rice-mustard cropping system consisting of a small RWST with a capacity of 61 m^3 and a command area of 800 m^2. Makurira et al. (2007) analyzed water distribution in a community-driven smallholder irrigation scheme including several RWSTs whose capacities range from 200 to 1600 m^3. Chang et al. (2011) proposed an optimal design scheme for roof RWH systems under mixed uncertainties stemming from irregular precipitation and water demand.

* Corresponding author.
E-mail address: unami@adm.kais.kyoto-u.ac.jp (K. Unami).

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Conventional hydrology considers runoff processes in a catchment area as the input–output relationship generating the runoff discharge from the precipitation (Nápoles-Rivera et al., 2013). Karczewska and Lizama (2009) presented stochastic Volterra equations applicable to such precipitation-runoff models. Unami and Kawachi (2005) identified runoff processes in a tank irrigated paddy fields area as transfer functions in the frequency domain.

An alternative novel approach is to model the output runoff exclusively, so that occurrence of recharge events filling the RWST of a RWH system is well reproduced. In this context, the Langevin equation, which is a stochastic differential equation, is employed here for representing alteration of recharge events and dry spells. The Langevin equation is coupled with a water balance equation of the RWST to construct a mathematical model for the RWH system. The model parameters are identified from time series data of recharge events and dry spells. Then, the stochastic control theory is further applied to this model, to constitute a feed-back control system determining the optimal irrigation strategies during a dry spell. The feed-back rule is obtained as a solution of the Hamilton-Jacobi-Bellman (HJB) equation, which is a second order partial differential equation with advection terms. A computational method including finite element and finite difference schemes is presented and applied to resolution of the HJB equation. Indeed, few researchers are tackling HJB equations with computational approaches. Boulbrachene and Haiour (2001) presented error estimate for finite element approximation of steady HJB equations with Dirichlet boundary conditions, and Boulbrachene and Dumont (2009) dealt with the case of Neumann boundary conditions establishing a quasi-optimal error estimate. Boulbrachene and Chentouf (2004) discussed the piecewise linear approximation of HJB equations with noncoercive operators. Kumar and Muthuraman (2004) illustrated optimal strategies for real world problems by numerically solving a class of singular stochastic control problems. They combined the finite element method for partial differential equations with a policy update procedure based on the principle of smooth pasting, in order to iteratively solve HJB equations.

Validity of the methodology proposed here is discussed in terms of demonstrative examples. Firstly, the identification procedure for the values of model parameters is examined with time series data observed at two contrasting sites: an experimental micro RWH system under temperate climate and a potential RWH system for dry season irrigation in a savanna zone. Then, a RWH system operated with optimal irrigation strategies is practically designed in Jordan Rift Valley, involving several issues. A dominant advection term of the HJB equation requires fine meshes for accurate solutions. Considering tradeoff between accuracy and computational costs, optimal irrigation strategies are computed under different conditions and represented in terms of rule curves prescribing water withdrawal limits. Rule curves for optimal operation of large irrigation dams have been well discussed in the literature. Senga (1991) defined rule curves for Japanese irrigation dams to anticipate future droughts, asserting their advantages over conventional management strategies. Moghaddasi et al. (2013) proposed rule curves for long-lead reservoir operation, considering regional optimal allocation of water among different crops and irrigation units. Khan et al. (2012) improved conventional rule curves for the Tarbela Reservoir, Pakistan, to minimize irrigation deficits allowing for sediment evacuation. Eum et al. (2012) developed an integrated reservoir management system to adapt operational strategies to changing climate conditions, generating the optimal reservoir rule curves for historic, dry, and wet climate scenarios. Genetic Algorithms (GAs) have been applied for determining rule curves set at different reliability levels (Jothiprakash and Shanthi, 2006), based on floods and water shortages scenarios (Suiadee and Tingsanchali, 2007), or constrained with a penalty strategy (Ngoc et al., 2014). However, there is no attempt dealing with a RWH system in the framework of stochastic calculus so far. It is demonstrated that the computational results generating the rule curves as a key tool strongly support the decision making process in practical irrigation water management in the RWH system.

2. Mathematical model and control system

A mathematical model for RWH systems is developed in this section. The identification procedure for the values of model parameters is described as well. Then, stochastic optimal control problems are formulated in order to synthesize optimal irrigation strategies. Computational methods are proposed to numerically solve the partial differential equations appearing in the methodology. The notation $H_0^1(\Omega)$ is used for representing the Sobolev space, which is the complete normed space of functions having certain regularity properties and vanishing on the boundary, on a generic domain Ω.

2.1. Model description

Dynamics of the storage volume X_t of the RWST at the time t is governed by

$$\mathrm{d}X_t = -u\mathrm{d}t \tag{1}$$

where u is the discharge of water outflowing from the RWST during a dry spell between two consecutive recharge events. The Langevin equation describes the dynamics of a zero-reverting stochastic process Y_t as

$$\mathrm{d}Y_t = -Y_t\mathrm{d}t + \sqrt{2}\mathrm{d}B_t \tag{2}$$

where B_t is the standard Brownian motion (Øksendal, 2007). The stochastic process Y_t is utilized for modeling the occurrence of recharge events and dry spells with a constant parameter K, by assuming that $|Y_t| < K$ during a dry spell and that $|Y_t| \geq K$ when the whole RWH system is so wet to recharge its RWST. However, defining another constant parameter K_τ ($>K$), a substantial recharge event of the RWST is assumed to take place only if $|Y_t| = K_\tau$ is achieved during a period of $|Y_t| \geq K$. The discharge u, which includes the flow rate of irrigation as well as the loss due to evapotranspiration and leakage, is considered as the control variable. When information of the stochastic process Y_t is fed-back to determine the flow rate of irrigation, the control variable u and then the storage volume X_t become stochastic.

2.2. Identification of model parameters

Mathematical principles are applied for identifying the values of the different model parameters from observed data. A recharge event R_i is defined as the period starting at $t = T_{2i-1}$ and ending at $t = T_{2i}$ for an integer i. A dry spell is the period from T_{2i} through T_{2i+1}.

The transition probability $P(0,y_0,t,G)$ of Y_t from time 0 to time t (>0) is the probability such that Y_t is in any subset G of $\mathbb{R}$ at time t provided that $Y_0 = y_0$. The probability density function $p(0,y_0,t,y)$ is defined so as to satisfy

$$P(0, y_0, t, G) = \int_G p(0, y_0, t, y)\mathrm{d}y. \tag{3}$$

For a fixed y_0, the probability density function $p = p(t,y) = p(0,y_0,t,y)$ is governed by the forward Kolmogorov equation