Journal of the Energy Institute 87 (2014) 69–80

Contents lists available at ScienceDirect

Journal of the Energy Institute

journal homepage: http://www.journals.elsevier.com/journal-of-the-energy-institute

Optimal performance of a spin quantum Carnot heat engine with multi-irreversibilities

X.W. Liu [a,b,c], L.G. Chen [a,b,c,*], F. Wu [c,d], F.R. Sun [a,b,c]

[a] Institute of Thermal Science and Power Engineering, Naval University of Engineering, Wuhan 430033, China
[b] Military Key Laboratory for Naval Ship Power Engineering, Naval University of Engineering, Wuhan 430033, China
[c] College of Power Engineering, Naval University of Engineering, Wuhan 430033, China
[d] School of Science, Wuhan Institute of Technology, Wuhan 430074, China

A R T I C L E I N F O

Article history:
Received 18 February 2011
Accepted 21 May 2013
Available online 18 February 2014

Keywords:
Finite time thermodynamics
Spin-1/2 systems
Quantum heat engine cycle
Power
Efficiency

A B S T R A C T

By using quantum master equation, semi-group approach and finite time thermodynamics (FTT), this paper derives the expressions of cycle period, power and efficiency of an irreversible quantum Carnot heat engine with irreversibilities of heat resistance, internal friction and bypass heat leakage, and provides detailed numerical examples. The irreversible quantum Carnot heat engine uses working medium consisting of many non-interacting spin-1/2 systems and its cycle is composed of two isothermal processes and two irreversible adiabatic processes. The optimal performance of the quantum heat engine at high temperature limit is deduced and analyzed by numerical examples. Effects of internal friction and bypass heat leakage on the optimal performance are discussed. The endoreversible case, frictionless case and the case without bypass heat leakage are also briefly discussed.

1. Introduction

In the past few decades, tremendous progress has been made in optimizing the performance of thermodynamic processes and cycles by using the theory of finite time thermodynamics (FTT) [1–8]. However, for some special fields and systems, such as, magnetic system, infrared techniques, laser system, superconductivity system, and so on, the working medium in these systems has quantum characteristics and obeys quantum statistical mechanics instead of classical statistical mechanics. Therefore, the classical thermodynamics based on phenomenological law and classical statistical mechanics based on equilibrium statistical mechanics are inapplicable. By considering the quantum characteristics of working medium, some researchers have applied the FTT to analyze and optimize the performance of quantum thermodynamic processes and cycles, and obtained many novel results. Kosloff [9] first established a quantum heat engine model and derived the power and efficiency of the quantum heat engine. This engine was called quantum harmonic engine and consisted of three parts: (1) the engine: the engine is similar to a quantum amplifier. It consists of two harmonic oscillators with different frequencies and produced power through a population inversion between the harmonic oscillators; (2) the power output mechanism: the power output mechanism is accomplished by a periodic coupling which manipulated the population difference between the oscillators; (3) the heat reservoirs: there are two heat reservoirs coupled the harmonic oscillators, respectively, and the reservoirs are describe by a semi-group approach. Geva and Kosloff [10] established an endoreversible quantum heat engine model and analyzed the optimal performance of the engine by using semi-group approach and FTT. This quantum heat engine is similar to the quantum harmonic engine in Ref. [9]. Differently, this quantum engine uses working medium consists of non-interacting spin-1/2 systems and is called quantum spin-1/2 heat engine. In the quantum spin heat engine, the time dependence of the external driving field is controllable, and the spin medium is carried along a Carnot cycle, which is composed of two isothermal branches and two adiabatic branches, by changing the magnitude of the external driving field over time. Geva and Kosloff [11] compared the performance of endoreversible spin quantum Carnot heat engine and endoreversible harmonic quantum Carnot heat engine, and concluded that the optimal thermodynamic pass of these two kinds of quantum heat engines are not Carnot type by using optimal control theory. Besides quantum Carnot cycle, Feldmann et al. [12] established an

---

* Corresponding author. Naval University of Engineering, College of Power Engineering, Wuhan 430033, China. Tel.: +86 27 836 15046; fax: +86 27 836 38709.
E-mail addresses: lingenchen@hotmail.com, lgchenna@yahoo.com (L.G. Chen).

http://dx.doi.org/10.1016/j.joei.2014.02.008
1743-9671/

List of symbols

a	parameter of heat reservoir (s^{-1})
B	heat reservoir
$\overrightarrow{B}$	external magnetic field (T)
C_e	dimensionless factor that describes the magnitude of the bypass heat leakage
c	parameter of heat reservoir (s^{-1})
E	internal energy of the spin-1/2 systems (J)
$\widehat{H}$	Hamiltonian
$\hbar$	reduced Planck's constant (J s)
k_B	Boltzmann constant ($J\ K^{-1}$)
L_1, L_2	Lagrangian functions
$\widehat{M}$	magnetic moment operator
m	intermediate variable
n_c	population of the thermal phonons of the cold reservoir
P	power (W)
Q	amount of heat exchange (J)
$\widehat{Q}_\alpha$, $\widehat{Q}_\alpha^+$	operator in the Hilbert space of the system and Hermitian conjugate
Q'	amount of heat exchange between heat reservoir and working medium (J)
$\dot{Q}$	rate of heat flow (W)
q	parameter of heat reservoir
S	expectation value of spin operator $\widehat{S}_z$
$\widehat{S}_+$, $\widehat{S}_-$	spin creation and annihilation operators
$\widehat{S}(\widehat{S}_x, \widehat{S}_y, \widehat{S}_z)$	spin operator
S_{eq}	asymptotic value of S
T	absolute temperature (K)
T'	absolute temperature of the working medium (K)
t	time (s)
W	work (J)

Greek symbols

α	intermediate variable
β	"temperature" $\beta = 1/(k_B T)$ (J^{-1})
β'	"temperature" of working medium $\beta' = 1/(k_B T')$ (J^{-1})
γ_+, γ_-	phenomenological positive coefficients
η	efficiency
λ	parameter of the heat reservoir
λ_1, λ_2	Lagrangian multipliers
μ	friction coefficient
μ_B	Bohr magneton ($J\ T^{-1}$)
$\widehat{\Gamma}$	interaction strength operator
τ	time (s)/cycle period (s)
ω_c	frequency of the thermal phonons (s^{-1})

Subscripts

B	heat reservoir
c	cold side
h	hot side
S	working medium system
SB	interaction between heat reservoir and working medium system
$\mu = 0$, $C_e = 0$	maximum point for endoreversible case
0	environment
1, 2, 3, 4	cycle states

endoreversible spin quantum Brayton heat engine cycle model and investigated its optimal performance, while Wu et al. [13] established endoreversible forward and reverse harmonic quantum Stirling cycle models and investigated the optimal performance of these quantum Stirling cycles. Wu et al. [14] investigated the optimal exergoeconomic performance of an endoreversible harmonic quantum Stirling engine. Lin and Chen [15] established an endoreversible harmonic quantum Brayton heat engine model and investigated the optimal performance of the quantum heat engine by using numerical solutions. Chen [16] investigated the optimal ecological performance of an endoreversible spin quantum heat engine, and the quantum heat engine cycle is composed of an adiabatic process, an isomagnetic field process and two isothermal processes.

Similar to analysis and optimization of heat engine with classical working medium, various sources of irreversibility, such as the heat resistance, bypass heat leakage, dissipation processes inside the working medium, etc., were considered in the analysis and optimization of quantum heat engines. Jin et al. [17] introduced a bypass heat leakage in the investigation of the optimal exergoeconomic performance of an irreversible harmonic quantum Carnot engine. The bypass heat leakage arises from the thermal coupling action between the hot reservoir and cold reservoir. Feldmann and Kosloff [18] introduced an internal friction into the investigation of the optimal performance of an irreversible spin quantum Brayton heat engine and heat pump. The internal friction describes the effects of non-adiabatic phenomenon, which arises from the rapid change of the external magnetic field, in the adiabatic process. Since then, origin of quantum friction and effects of it on the performance of irreversible quantum thermodynamic cycles have attracted a lot of attentions [19–26]. Lin and Chen [27] investigated the optimal performance of an irreversible harmonic quantum Stirling heat engine by taking into account irreversibilities of heat resistance and inherent regeneration. Wu et al. [28,29] established generalized irreversible harmonic [28] and spin [29] quantum Brayton heat engine models with heat resistance, internal irreversibility and bypass heat leakage, and investigated the optimal performance of the two quantum heat engines. Different from the internal friction in Ref. [18], internal irreversible factors ϕ were used to describe the irreversibility inside the irreversible adiabatic processes in the two quantum heat engine cycles. Wu et al. [30] established a generalized irreversible spin quantum Carnot heat engine model with heat resistance, internal irreversibility (described by an internal irreversible factors ϕ) and bypass heat leakage. Different from the works mentioned above, a new function, the Helmholtz free energy of a two-level system, was used to calculate the heat exchanges between the working medium and heat reservoir in Ref. [30]. Liu et al. [31] established a generalized irreversible harmonic quantum Carnot heat engine model with heat resistance, internal friction and bypass heat leakage, and investigated the optimal ecological performance of the quantum heat engine. The irreversibility of non-adiabatic phenomenon was described by internal friction coefficient and that was different from the internal irreversible factor used in Refs. [28–30].

In the performance analysis and optimization of quantum heat engines which use harmonic or spin working medium, the thermal coupling between the working medium and the heat reservoir is described by the Hamiltonian or Liouvilles's operator, and the motion equation of arbitrary operator the working medium system is given by the quantum master equation and semi-group approach. At weak coupling limit and high temperature limit, the thermodynamic quantities and the performance parameters of the engine cycle are obtained