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Review of power curve modelling for wind turbines

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ABSTRACT

Currently, variable speed wind turbine generators (VSWTs) are the type of wind turbines most widely installed. For wind energy studies, they are usually modelled by means the approximation of the manufacturer power curve using a generic equation. In literature, several expressions to do this approximation can be found; nevertheless, there is not much information about which is the most appropriate to represent the energy produced by a VSWT. For this reason, in this paper, it is carried out a review of the equations commonly used to represent the power curves of VSWTs: polynomial power curve, exponential power curve, cubic power curve and approximate cubic power curve. They have been compared to manufacturer power curves by using the coefficients of determination, as fitness indicators, and by using the estimation of energy production. Data gathered from nearly 200 commercial VSWTs, ranging from 225 to 7500 kW, has been used for this analysis. Results of the analysis presented in the paper show that exponential and cubic approximations give the higher R^2 values and the lower error in energy estimation. With the approximate cubic power curve quite high values of R^2 and low errors in energy estimation are achieved, which makes this kind of approximation very interesting due to its simplicity. Finally, the polynomial power curve shows the worst results mainly due to its sensitivity to the data given by the manufacturer.

Contents

1. Introduction

The power curve of a WTG is obtained by the manufacturers from field measurements of wind speed and power, apart from environmental values (temperature, pressure and relative

Abbreviations: WTG, wind turbine generator; VSWT, variable speed wind turbine

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Nomenclature

v	wind speed in m/s
A	rotor area in m^2
ρ	air density in kg/m^3
$p(v)$	electric power generated by the wind turbine in W
C_p	power coefficient
$p_w(v)$	power in W associated to a wind speed
v_{ci}	cut-in wind speed in m/s
v_{co}	cut-out wind speed in m/s
v_r	rated wind speed in m/s
P_r	rated power in W
$q(v)$	non-linear part of a power curve
$q'(v)$	non-linear part of fitted power curve
E	energy density in W/m^2
E'	energy density obtained from fitted curve
$f(v)$	probability in p.u. associated to a wind speed v
N	number of values in manufacturer power curve
v_I	discretised value of cut-in speed
v_O	discretised value of cut-out wind speed
v_R	discretised value of rated wind speed
f_{rj}	relative frequency associated to each wind speed v_j
C_1, C_2, C_3	coefficients of polynominal approximations
K_p, β	coefficients of exponential approximation
$C_{p,eq}$	coefficient of cubic approximation
$C_{p,max}$	maximum value of effective power coefficient
J	index for least square optimisation
$\xi\{\cdot\}$	mean function
R^2	coefficient of determination
ε	error of energy density in %

humidity). The measurements are usually averaged and normalised to a reference air density using normalised procedures [1]. The resulting discrete values of the power curve for a determined WTG are usually available from manufacturers, and they can be used for studies involving energy evaluation.

Nevertheless, for the sake of generality, it is common that a generic equation for modelling the power curve will be preferred in studies about WTG modelling [2–5], analysis of wind energy potential [6], site matching [5,7–9], cost modelling [10,11], etc. In this context, the use of an equation for representing a power curve and the obtention of its parameters becomes an important issue. The main problem derived from using a generic equation is the fact that is hard to know how this equation will accurately represent any commercial WTG.

In the first term, the power curve of a WTG can be estimated using the power curve coefficient (C_p) from the turbine blade parameters (blade design, tip speed ratio and pitch angle) [4], the rotor dimensions and the reference air density. For example in [12] the power coefficient is calculated through an expression that links the blade radius, blade design constant and wind turbine shaft angular speed with the power coefficient. In [11] an expression is proposed for the approximation of C_p, considering a rated power coefficient, rated wind speed and a parameter expressing the operation range of wind speed. The shortcomings of using the models proposed in [11] and [12] are that they depend on some technical factors of the wind turbines which are difficult to obtain from the manufacturers.

Another way to approximate the power curve is presented in [2], where power curves are approximated by means of fitting techniques, like least squares or cubic spline interpolation. Although pretty accurate fits are achieved, the resulting power curve equations are quite complex, which makes it difficult to find a generic expression.

To overcome the problems depicted above, the power curve of WTGs is usually represented by means of a polynomial power curve [13–15] or by means of an exponential power curve or its simplifications [16]. Their parameters can be derived from manufacturer data or by fitting the manufacturer power curve. However, although these expressions are widely used, there is little evidence of how these curves fit with real WTGs [13–18]. For this reason, in this paper is presented a study of the power curve models taking into account a database with manufacturer information from nearly 200 variable speed wind turbines (VSWT). Only VSWTs have been considered in this paper because they represent the state of art of commercial WTGs installed at present. The most important wind turbine manufacturers have been included in this database.

In order to analyse which are the most appropriate equations to approximate power curves, it is also presented a critical comparison of the fitted power curves considering the coefficient of determination R^2, as a measure of goodness of fit, and the difference between the estimations of energy density when the fitted and the manufacturer power curves are used.

The paper is organised as follows. Section 2 presents the identification of the main features of the power curve. Section 3 summarises the most typical models used for the representation of the power curves. Section 4 shows the database used for the characterisation of the power curves including the main characteristics of the wind turbines. In Section 5, the results of the fitting methods and indicators of fitness are presented. Finally, conclusions are given in Section 7.

2. Energy evaluation and power curve

The available power of the wind that crosses the rotor of a wind turbine can be obtained from

$$p_w(v) = \tfrac{1}{2}A\rho v^3 \quad (1)$$

where $p_w(v)$ is the power in W associated to a wind speed v in m/s, A is the rotor area in m^2 and ρ is the air density (typ. 1225 kg/m^3 [1]). This power is related to power generated by a wind turbine by means of the power coefficient

$$C_p(v) = p(v)/p_w(v) \quad (2)$$

where $p(v)$ is the power generated by the wind turbine in W, C_p is the power coefficient that is related to the blade design, the tip angle and the relationship between rotor speed and wind speed. The maximum theoretical value of power coefficient, known as the Betz limit, is 0.593 (16/27). However, this value is not achievable with real turbines and its maximum value is normally around 0.5. The power coefficient can be obtained from the manufacturer data, as a consequence, mechanical and electrical losses are usually included in the coefficient value as well as the aerodynamic behaviour of blades.

The power delivered by a wind turbine is usually represented through its power curve, where a relation between the wind speed and the power is established. For the VSWTs, this relationship can be expressed in the following way:

$$p(v) = \begin{cases} 0 & v < v_{ci} \ \ or \ \ v > v_{co} \\ q(v) & v_{ci} \le v < v_r \\ P_r & v_r \le v \le v_{co} \end{cases} \quad (3)$$