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Near-exponential relationship between effective stress and permeability of porous rocks revealed in Gangi's phenomenological models and application to gas shales

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ABSTRACT

A number of theoretical models, as well as empirical equations obtained by fitting specific experimental data, have been developed to describe the relationship between effective stress and permeability of intact and fractured porous rocks. It has been found that most experimental data can be fitted using exponential equations. In this study the modified power law equations by Gangi for intact and fractured rocks are revisited to evaluate their applicability for modeling experimental permeability data which display exponential or near-exponential effective stress dependency. It has been shown that Gangi's power law equations for both intact and fractured rocks can be approximated, over the range of effective stresses of practical interest, by exponential equations with compressibilities that are related to the physical properties of the rock. The significance of this work is that it has provided further theoretical evidence for the apparent exponential relationship between effective stress and permeability. Moreover, it allows for more vigorous theoretical equations to be applied with the easiness of empirical exponential equations. This is demonstrated by applying the models to the experimental permeability data for six gas shales reported recently.

1. Introduction

Experimental studies on the variation of intact porous rock permeability with effective stress applied on the rock have been reported in the literature since the 1950s (e.g. Fatt and Davis, 1952; Dobrynin, 1962; Gray et al., 1963; Knutson and Bohor, 1963; Durucan and Edwards, 1986; Kwon et al., 2001; Bustin et al., 2008; Dong et al., 2010; Heller et al., 2014). Empirical equations have been proposed to describe the experimental permeability data. They include exponential functions (e.g., Louis et al., 1977), power functions (e.g., Tiller, 1953; Kranz et al., 1979) and logarithmic functions (e.g. Dobrynin, 1962; Jones and Ovens, 1980).

As well as empirical equations, theoretical models have also been developed to describe the experimental data and predict the permeability behavior under stress. Gangi (1978) developed a phenomenological model, based upon the Hertzian theory of deformation of spheres, to predict the variation with effective stress with the permeability of intact porous rock. Gangi noted that if a porous rock is assume to be made up of a packing of spherical grains of uniform size, then its permeability will be a function of the square of the cross-sectional area of its pores. Gangi further pointed out that permeability of the porous rock also depends upon the number of pore channels in a unit area perpendicular to the direction of flow and the number of pore channels per unit area is inversely proportional to the square of the sphere radius. Thus; if r_p is the pore radius and R is the spherical grain radius (see Fig. 1a), the permeability of the porous rock in consideration will be given by

$$k \propto \frac{r_p^4}{R^2}. \tag{1}$$

Hertz theory gives change in separation, α, between sphere centers in a sphere packing due to an external force F, as (Fig. 2):

$$\frac{\alpha}{2R} = \left(\frac{d}{R}\right)^2 = \left[\frac{3(1-\upsilon^2)FR}{4E}\right]^{2/3} \Big/ R^2 = \left[\frac{3\pi(1-\upsilon^2)P}{4E}\right]^{2/3} = \left(\frac{P}{K_{gra}}\right)^{2/3} \tag{2}$$

where:

α	the "approach" (or change in separation) of the sphere centers;
R	the radius of the spherical grains;
d	the radius of the circle-of-contact of the spheres;
F	the external force;
E	the Young's modulus for the grains;
υ	the Poisson's ratio for the grains;

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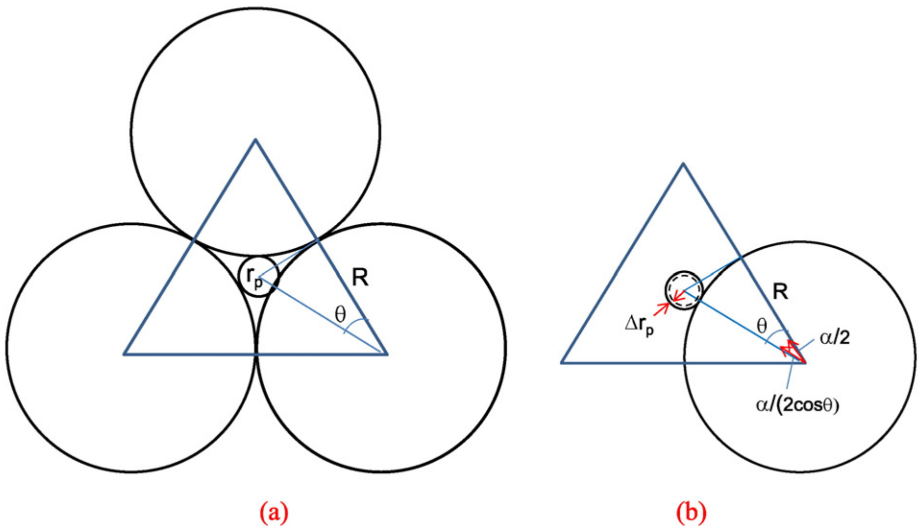

Fig. 1. a) A schematic of uniform grain packing; b) reduction in the pore radius caused by grain deformation (modified from Gangi, 1978).

P the external stress $= F/\pi R^2$;
K_{gra} the effective elastic modulus for the grains given by

$$K_{gra} = \frac{4E}{3\pi(1-\upsilon^2)} = \frac{4K(1-2\upsilon)}{\pi(1-\upsilon^2)}$$

where K is the bulk modulus for the grains. Gangi noted that K_{gra} is typically of the same order of the grain material bulk modulus ($K_{gra} \approx 0.7K$ for $\nu \approx 0.25$).

Gangi assumed, to a first approximation, that the pore shape does not change significantly (except in size) as P is increased and obtained the following expression for the reduction in the "pore radius" (Fig. 1b)

$$\Delta r_p = c_1 \alpha \cong 1/(2\cos\theta)\alpha. \tag{3}$$

Upon using Hertz's theory Eq. (2), Eq. (3) may be rearranged as

$$\Delta r_p = r_p \left(\frac{2c_1 R}{r_p}\right)\left(\frac{\alpha}{2R}\right) = r_p C_0 \left(\frac{P}{K_{gra}}\right)^{2/3} \tag{4}$$

where $C_0 = 2c_1R/r_p$ is a constant depending upon the packing. Combining Eqs. (1) and (4), the following equation is obtained by Gangi

$$k_{wr} = k_{lp}\left[1 - C_0\left(\frac{P}{K_{gra}}\right)^{\frac{2}{3}}\right]^4 \tag{5}$$

where k_{lp} is the initial permeability of the loose-grain packing. Gangi recognized that in real rocks, there will be some cementation or permanent deformation of the grain contacts such that the radius (d in Fig. 2) of the area-of-contact is not zero at zero stress. He pointed out that this is equivalent to having some initial stress, P_i, acting on the rock and modified the equation accordingly,

$$k_{wr} = k_{lp}\left[1 - C_0\left(\frac{P+P_i}{K_{gra}}\right)^{\frac{2}{3}}\right]^4. \tag{6}$$

Although Eq. (6) is derived based upon the assumption that the grains are all the same size spheres. Gangi argued that the same functional dependence would be expected even for a distribution of grain sizes. Gangi also pointed out that the underling Hertzian theory Eq. (2) is unlikely to be valid for the effective stress P_e above 0.03 K_{gra}. However, given that K_{gra} is typically of the order of the grain bulk modulus, this would correspond, in general, to stresses as high as 10^5 psi (~69 MPa), well above the range of most measurements.

The response of fracture permeability in rocks to applied effective stress has also attracted a lot of interests from the research communities since the 1970s (e.g. Jones, 1973; Nelson, 1975; Witherspoon and Gale, 1977). Both empirical equations and theoretical models have been developed to describe the experimental data and predict the fracture permeability behavior. It is commonly assumed in the derivation of the theoretical models that the fracture permeability is proportional to the cubic of its aperture.

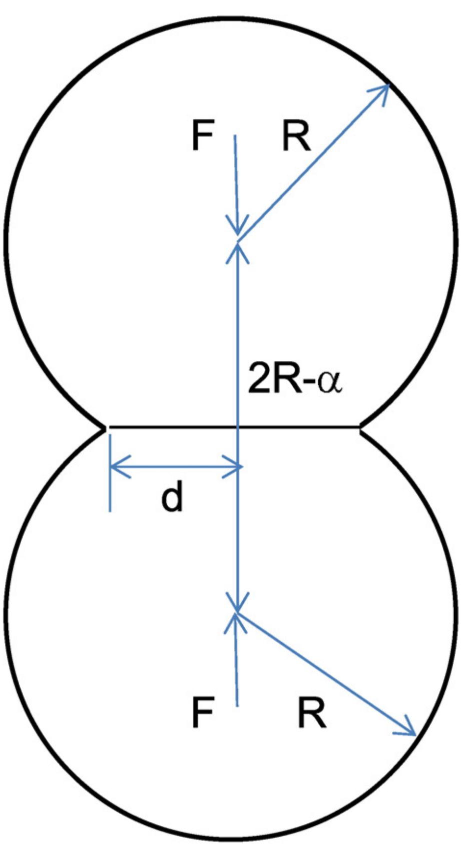

Fig. 2. Sphere–sphere deformation considered in Hertz theory (after Gangi, 1978).