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An analytical model for shale gas permeability

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ABSTRACT

Based on the kinetic theory of gases and using the regularized 13-moment method, the analytical R13 AP model is introduced for predicting gas apparent permeability of nanoporous shale samples. These samples are characterized by ultratight pores and may introduce significant rarefaction effects, especially under the laboratory conditions, which cannot be accounted for in the classical hydrodynamic equations. Due to the significance of the rarefaction effects, measured values of the gas apparent permeability depend on the operating parameters, such as pressure and temperature, and gas type in addition to pore size. The R13 AP model incorporates these parameters and can predict the apparent permeability for Knudsen numbers up to unity. This model is compared with the other models and experimental results. The results of the R13 AP model match the published experimental data of flow in nanochannels. It is shown that the gas molecular weight and temperature have a significant effect on apparent permeability of the nanochannels, and the Tangential Momentum Accommodation Coefficient (TMAC) has a minimal effect for its experimental range (0.8–1). The effect of adsorption on apparent permeability of nanochannels is studied by employing the experimental Langmuir isotherms of different shale samples. It is shown that the apparent permeability does not change linearly with surface coverage and, change of permeability with surface coverage becomes more pronounced as the surface coverage increases. R13 AP model results for apparent permeability of Carbon Dioxide and Nitrogen agree with the experimental measurements performed on a Marcellus shale sample. The absolute permeability values are calculated and compared with the ones estimated by the double slippage model for both gases. Unlike the double slippage model, the R13 AP model honors the values of apparent permeability at high pressure and estimates lower absolute permeability values. For Carbon Dioxide, the value is slightly lower due to the presence of an adsorbed layer.

1. Introduction

The United States' Natural gas production from shale resources increased from 4% of total gas production in 2005 to 40% in 2012, and is expected to rise to 53% in 2040 (U.S. Energy Information Administration, 2014). These resources are different from conventional hydrocarbon resources due to the presence of extremely tight organic pores and low permeabilities (Civan et al., 2010; Javadpour, 2009; Loucks et al., 2012). The presence of the nanopores may cause rarefaction effects, especially in laboratory conditions, which pronounces the effects of temperature and pressure on the apparent permeability of shale samples (Chalmers et al., 2012; Darabi et al., 2012; Passey et al., 2010). In order to determine the permeability of these resources, laboratory measured apparent permeabilities, if conducted at low pressures and temperatures, need to be extrapolated to reservoir conditions. In addition, gas flow at low pressure has important applications in predicting the gas production rates from shale gas reservoirs. Therefore, it is essential to develop accurate descriptive transport models for nanoporous rocks, which are capable of predicting rarefied flow dynamics.

The aim of this study is to develop an analytical model for the apparent gas permeability in two dimensional nanochannels. To do this, the regularized 13 (R13)-moment equations, which were developed by Struchtrup and Torrilhon (2003) and linearized by Taheri et al. (2009), are solved to find an analytical model for apparent permeability. The R13 AP model is compared with the linearized Boltzmann kinetic data and shown to be valid for Knudsen numbers up to unity in contrast to the existing analytical models (Azom et al., 2012; Civan, 2010; Javadpour, 2009; Singh et al., 2014). In this model, gas molecular weight and temperature have a significant effect on gas apparent permeability at low pressure, which is contrary to the previous models (Javadpour, 2009; Singh et al., 2014). In addition, the effect of adsorption on apparent permeability of shale samples is addressed by employing the experimental Langmuir isotherms measured by Heller and Zoback (2014).

In tight shale pores, gas flow regimes are characterized by the Knudsen number, *Kn*, which is the ratio of the gas mean free path to the characteristic length of the media. The mean free path of a particular gas depends on the molecular velocity and the molecular size which are nonmeasurable quantities. One approach to calculate the mean free

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path is to use its relationship with transport coefficients provided by the kinetic theory of gases as,

$$\lambda = \frac{\mu}{P}\sqrt{\frac{\pi\theta}{2}} \tag{1}$$

where $\theta = RT$, R is the specific gas constant, and T is the temperature. One of the advantages of this formulation is that it contains quantities that can be easily measured (Sharipov and Seleznev, 1998). Flow regimes consist of continuum flow ($0 < Kn < 10^{-3}$), slip flow ($10^{-3} < Kn < 10^{-1}$), transition flow ($10^{-1} < Kn < 10$), and free molecular flow ($10 < Kn < \infty$) (Roy et al., 2003) (Fig. 1). The Navier–Stokes equations describe the fluid flows with small Knudsen numbers ($Kn < 0.01$) (Struchtrup, 2005). In organic nanopores in shale, the Knudsen number is larger than 0.1 (Kang et al., 2011), which causes significant rarefaction effects. Thus, calculation of the permeability based on the continuum assumptions can lead to inaccurate results.

Gas flow through micro- or nanochannels has been previously studied by several authors using the slip boundary conditions (Azom et al., 2012; Civan, 2010; Darabi et al., 2012; Javadpour, 2009). Javadpour (2009) used the diffusive–advective gas flow model to develop an analytical model for apparent permeability of the nanotubes. The total mass flux is determined by superposition of the Knudsen diffusion and advective gas flow driven by pressure forces. Knudsen diffusion term is derived by fully diffusive gas–wall interaction assumption (Mason and Malinauskas, 1983); in the pressure force term, on the contrary, gas–wall interactions can be adjusted by changing the Tangential Momentum Accommodation Coefficient (TMAC). Beskok and Karniadakis (1999) used the Navier–Stokes equations with second order slip boundary conditions and developed an equation for volumetric gas flow through a single pipe for $Kn < 0.5$ (unified flow model). Civan (2010) employed the Beskok and Karniadakis (1999) model to develop an equation for apparent gas permeability.

Several numerical and experimental models for apparent permeability have also been proposed (Akkutlu and Fathi, 2012; Civan, 2010; Florence et al., 2007; Jones et al., 1980; Sakhaee-Pour and Bryant, 2011; Sampath and Keighin, 1982; Tang et al., 2005; Zhu et al., 2009). The contribution of the surface roughness added to the Javadpour (2009) model by Darabi et al. (2012). The NAP model which was recently developed by Singh et al. (2014) is claimed to be a nonempirical model for ultratight porous media. In this model, the no slip flow is assumed to remove the apparent permeability dependency on Tangential Momentum Accommodation Coefficient (TMAC). However, the Knudsen diffusion term used in this model contains the assumption of fully diffusive boundary condition (TMAC = 1) (Mason and Malinauskas, 1983). In Section 3, it is shown that this model underestimates the mass flux in the transition regime ($Kn > 0.1$) because it neglects the significant contribution of gas slip to the total flux (Lockerby et al., 2005).

In this paper, analytical R13 AP is developed for gas apparent permeability of the two dimensional micro- and nanochannels. The developed model describes details of gas transport in nanochannels, such as tangential heat transfer in isothermal flow, and is valid for Knudsen numbers up to the unity. The model is validated with linearized Boltzmann kinetic data (Cercignani and Daneri, 1963). It is also shown that the R13 AP model agrees well with the results of the steady-state apparent permeability experiments performed on a Marcellus shale core plug. In addition, the effects of molecular weight, temperature, pressure, pore size, and TMAC on apparent gas permeability are investigated. The effect of gas adsorption on the gas apparent permeability of shale samples is also addressed by proposing a simple apparent permeability model. The remainder of this paper is outlined as follows. In Section 2, the mathematical model and boundary conditions are described. The analytical solution of the apparent permeability is derived and compared with the experimental data of Maurer et al. (2003). The velocity and mass flux profiles are presented and compared with linearized Boltzmann data of Cercignani and Daneri (1963) in Section 3.1. The impacts of the pressure, temperature, molecular weight, TMAC, and adsorption on the gas permeability are investigated in Section 3.3. In Section 3.4, the model is compared against the Marcellus shale experimental data. Summary and conclusions are provided in Section 4.

2. Mathematical model

In 1872, Boltzmann developed an equation describing the state of dilute gases and established a basis for the kinetic theory of gases. It describes the gas dynamics using the particle velocity distribution function,

$$\frac{\partial f}{\partial t} + \xi_k \frac{\partial f}{\partial x_k} + G_k \frac{\partial f}{\partial \xi_k} = \mathrm{J}(f, f) \tag{2}$$

where $f(x_k, t, \xi_k)$ is the particle distribution function, ξ_k is the microscopic particle velocity, G_k is the external force, and $\mathrm{J}(f, f)$ describes the change of phase density due to collision between particles. Macroscopic

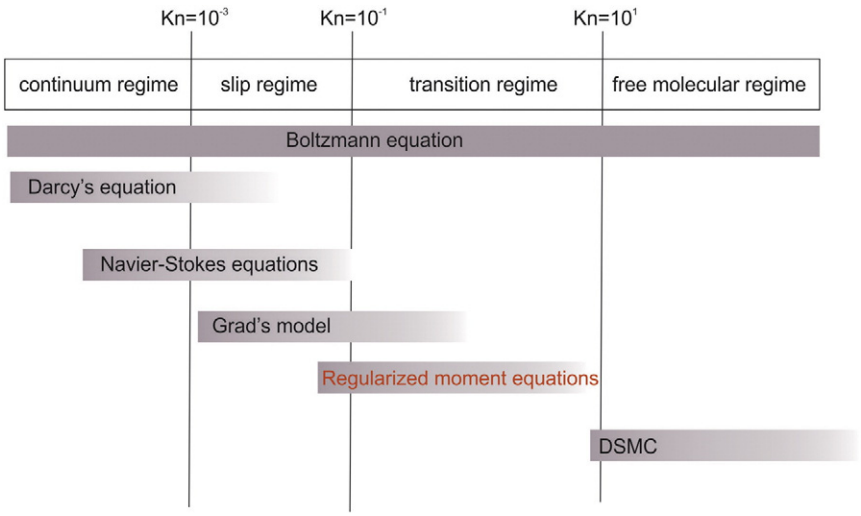

Fig. 1. Flow regimes at different Knudsen numbers.