International Journal of Coal Geology 112 (2013) 14–25

Contents lists available at SciVerse ScienceDirect

International Journal of Coal Geology

journal homepage: www.elsevier.com/locate/ijcoalgeo

Geostatistical estimation of coal quality variables by using covariance matching constrained kriging

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ARTICLE INFO

Article history:
Received 30 June 2012
Received in revised form 28 November 2012
Accepted 28 November 2012
Available online 5 December 2012

Keywords:
Covariance matching constrained kriging
Variogram
Block kriging
Block model

ABSTRACT

Covariance matching constrained kriging is a technique originally developed for estimating linear or nonlinear functional of the variable of interest. Practically this may be regarded as a hybrid system that considers local accuracy property in kriging and preservation of spatial variability in stochastic simulation. The method estimates the unknown vector by adding the constraint to match the variance–covariance matrix of estimated values with the variance–covariance matrix of actual values under unbiasedness constraint.
This study applies the method to a certain section of a lignite deposit which is subjected to severe tectonic activity. Lower heating value, ash content, and moisture content are estimated by covariance matching constrained kriging and ordinary kriging and also simulated by sequential Gaussian simulation. Comparisons between these methods show that covariance matching constrained kriging reproduces the spatial variability better than ordinary kriging and gives more accurate estimates than conditional simulation. Covariance matching constrained kriging can be used when spatial variability of the estimations is important as well as local accuracy.

1. Introduction

Recoverable resources are that part of resources that grade value exceeds a given cutoff value. These resources are typically estimated for a given selective mining block and a number of cutoff values and presented by grade–tonnage curves. Recoverable resources are used in all phases of a mining project, including feasibility, mine planning and production scheduling and therefore should be accurately estimated.

The problem here is the one of estimating nonlinear functionals of block means. One way of doing this would be to estimate the mean grade of the selective mining blocks by kriging and to apply cutoffs to the estimated values. However kriging produces smoothed estimates with large values underestimated and small values overestimated. Conditional simulation offers solution for the nonlinear estimation problem, generating values that have the same spatial variability as the actual values. The price to pay for this is loss in accuracy and also biased estimates if the probabilistic model is misspecified.

By taking the advantages of these two techniques, Aldworth and Cressie (2003) proposed a hybrid technique called covariance matching constrained kriging (CMCK) to provide unbiased estimation of nonlinear functional of kriging estimates. The aim is to predict a nonlinear function such as indicator $I(Z(A)\geq t)=0$ and $I(Z(A)<t)=1$ with a given threshold t, based on data $Z\equiv(Z(s_1),\ldots, Z(s_n))'$ where $\{Z(s): s\in D\}$ and $Z(A)\equiv \mathrm{ave}\{Z(s):s\in A\}$ where A being a proper subset of a domain D. Disjunctive kriging (Matheron, 1976), indicator kriging (Journel, 1983), and indicator cokriging (Lajaunie, 1990) are nonlinear predictors of nonlinear functionals of Z, e.g. $I(Z(s_0)\geq t)$, but they are problematic when predicting $I(Z(A)\geq t)$. Conditional simulation (Deutsch and Journel, 1992) offers solution for this problem. CMCK predictor is built by adding second moment that is forcing elements of the variance matrix of predictors match with corresponding predictands (Cressie and Johannesson, 2001). Tercan (2004) applied CMCK to a lignite deposit in order to estimate the global recoverable reserve. The study also covers a comparison of CMCK with ordinary kriging (OK) on estimation of quality tonnage curves. By mimicking assessment of the pollution around a metal smelter, Hofer and Papritz (2010) examined the performances of various techniques such as universal kriging (UK), conditional simulation (CS), constrained kriging (CK) and CMCK. In the performance comparison estimations of block means (linear) and threshold exceedance of block means (nonlinear) were considered for four hexagonal blocks.

Hofer and Papritz (2011) developed an R-package "constrained kriging" which computes CK, CMCK and UK predictions for points or blocks of arbitrary shape from data observed at points in a two-dimensional survey domain.

The present study addresses a case study in which CMCK, OK and CS are applied to estimation and simulation of three coal quality variables [lower heating value (LHV), ash content (AC), and moisture content (MC)] in a lignite deposit located in the Ömerler sector of the Tunçbilek coalfield, Kütahya, Turkey. In the study the LHV, AC and MC means of the blocks with size of 25 m × 25 m × 1 m are estimated and simulated. OK and CMCK are used in estimation, sequential Gaussian simulation (sGs) in geostatistical simulation. Then the

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0166-5162/$ – see front matter
http://dx.doi.org/10.1016/j.coal.2012.11.014

quality-tonnage curves are derived from block estimations and simulations and they are compared. In comparisons histograms and variograms for the block means are also considered.

2. Methodology

Estimation criterion may be (1) local accuracy, (2) reproduction of spatial variability or (3) a mixture of both, depending on the problem at hand. In the case of OK the criterion is to estimate local value as accurately as possible. Instead conditional simulation focuses on reproduction of global statistics such as variogram and histogram, CMCK aims to produce estimates that are locally accurate and spatially variable. Fig. 1 shows a schematic representation of the link between these three methods.

The CMCK method estimates unknown values such that the variance–covariance matrix of estimated values is matched with the variance–covariance matrix of actual values under unbiasedness constraint.

$$\left(\mathrm{Var}\left[Z_v^{*}\right]+\mathrm{Var}\left[\mu_{gls}\right]\right)\times\mathbf{K}=\left(\mathrm{Var}[Z_v]+\mathrm{Var}\left[\mu_{gls}\right]\right) \quad (1)$$

where Z_v is the true block value, Z_v^* is the estimated block value, and $\mathrm{Var}[Z_v^*]$ and $\mathrm{Var}[Z_v]$ are variance–covariance matrices of estimated values and actual values respectively. μ_{gls} is mean estimated from generalized least square and $\mathbf{K}$ is the matrix that controls the matching process and is obtained from Eq. (2),

$$\begin{aligned}\mathbf{P}&=\left(\mathrm{Var}[Z_v^{*}]+\mathrm{Var}\left[\mu_{gls}\right]\right)=\mathbf{L}_P{}^{T}\mathbf{U}_P\\ \mathbf{Q}&=\left(\mathrm{Var}[\mathbf{Z}]+\mathrm{Var}\left[\mu_{gls}\right]\right)=\mathbf{L}_Q{}^{T}\mathbf{U}_Q\\ \mathbf{L}_P\mathbf{K}&=\mathbf{U}_Q\\ \mathbf{K}&=\mathbf{L}_P{}^{-1}\mathbf{U}_Q\end{aligned} \quad (2)$$

where $\mathbf{L}_P$, $\mathbf{U}_P$, $\mathbf{L}_Q$ and $\mathbf{U}_Q$ are roots of LU decomposition of $\mathbf{P}$ and $\mathbf{Q}$ respectively.

Then CMCK estimator becomes,

$$Z^*_{V(CMCK)}=\mu_{gls}+\mathbf{K}C(v-x_i)'C\left(x_i-x_j\right)^{-1}\left(Z(x_i)-\mu_{gls}\right) \quad (3)$$

where C_{vi}^{T} is a vector with elements of mean covariances between block v and location x_i, and C_{ij} is a matrix including covariances between locations x_i and x_j. T and −1 denote transpose and inverse respectively. Zi is a vector with elements of the measured values at location x_i, $i=1,\ldots,n$. Q and P matrices should be positive-definitive, otherwise CMCK predictor does not exist. When CMCK predictor does not exist, Aldworth and Cressie (2003) suggest partitioning the vector Z^* into sub-vectors, and then applying the matching in order to get positive definitive decompositions. Even if the matrix P is nonnegative, there will always be a practical limitation imposed by computer technology to the decomposition of the matrix P and therefore the number of points to be estimated, as it happens in LU geostatistical simulation method. In this study, matrices are partitioned into sub-matrices so that P matrix is non-positive-definitive. Kukush and Fazekas (2005) proposed a competing method to compute the CMCK estimators but it appears that it is tricky to implement it and the estimator of Kukush and Fazekas is less stable than Aldworth and Cressie's. Geostatistical tools such as variogram, kriging and conditional simulation are nicely presented in the paper by Srivastava (2013–this issue).

3. Case study

3.1. Field description and geological setting

The data used in this study come from the Ömerler sector of the Tunçbilek coalfield located in the western part of Turkey. Tunçbilek is a district of Tavşanlı–Kütahya and the Ömerler coalfield is located in the northern part of Tunçbilek.

The Tunçbilek basin is situated between Tunçbilek and Domaniç (Kütahya) in the northeastern part of a horst–graben system in western Turkey. The metamorphic and ophiolitic rocks and granites of the Pre-Neogene age form the basement of the basin (Fig. 2). The coal-bearing Domaniç Formation was developed in lacustrine facies (mudstone, claystone, coal, and marl), continental deltaic conglomerate–sandstone, continental fan deltaic conglomerate–sandstone–mudstone and lacustrine limestone. Overall thickness of the Miocene–Pliocene formations in the basin is greater than 1 km (Karayiğit and Çelik, 2003). An average 7 m thick coal bed lies at the base of the Domaniç Formation. The coal bed lies between marl and conglomerate–sandstone units and includes dirt bands as claystone with coal traces, marls, and alternations of coal and claystone (Karayiğit and Çelik, 2003).

Coal blocks, in general, strike N69W and are dipping N7E. The considered part of the coal seam spreads approximately an area of 1.5 km^2 and has 17 M m^3 volume. Fig. 3 represents the drill-hole locations and coal seam. Southern part of this coalfield has been mined out.

3.2. Compositing

In the area under consideration 245 vertical holes are drilled by various bodies. Core samples collected from these drill-holes are analyzed for LHV, AC and MC on an as-received basis. These raw samples had variable intervals with mean of 0.52 m and are composited to 1 m down-hole lengths. Since rock partings are one of the parameters taken into account in compositing, the minimum rock partings are accepted to be 0.50 m. LHV, AC, and MC of these partings are assumed to be 1 kcal/kg, 75%, and 25% respectively. Histograms of composites for three variables are presented in Fig. 4. LHV is of a left skewed

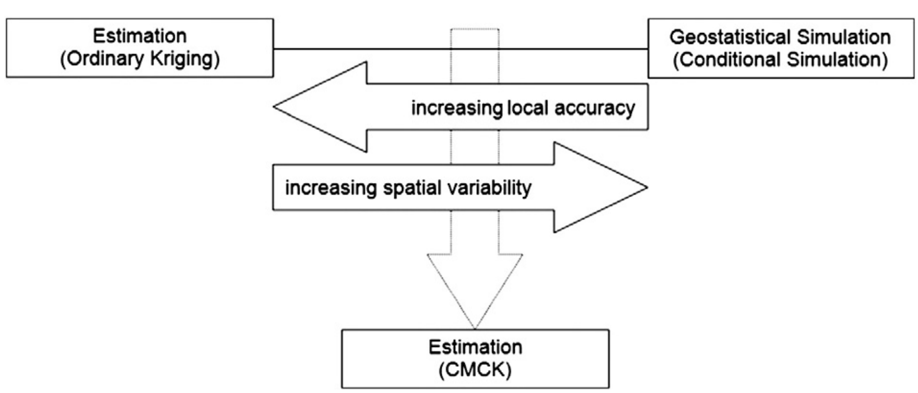

Fig. 1. A schematic representation of the link between the three methods.