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Geostatistical modelling of a coal seam for resource risk assessment

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ABSTRACT

The evaluation of a coal seam for profitable extraction requires the estimation of its thickness and quality characteristics together with the spatial variability of these variables. In many cases the only data available for the estimation are from a limited number of exploration and feasibility drill holes. Spatial variability can be quantified by geostatistical modelling, which provides the basis for estimation (kriging). In cases where the spatial variability of the seam thickness and quality characteristics has a significant impact on how the coal is extracted and stored, geostatistical simulation may be preferable to geostatistical kriging methods. The aim of this paper is to present an improved approach to resource risk assessment by propagating the uncertainty in semi-variogram model parameters into the spatial variability of coal variables. We show that a more realistic assessment of risk is obtained when the uncertainty of semi-variogram model parameters is taken into account. The methodology is illustrated with a coal seam from North-western Spain.

1. Introduction

One of the most significant contributors to the total risk in the evaluation of a coal-mining project is the uncertainty of the resource tonnage and quality characteristics, often called the resource risk (see, for example, Sobczyk, 2010). The depositional and tectonic history of basin coal deposits is a significant determinant of the spatial variability of the thickness and quality characteristics of constituent coal seams and may have a major impact on the accuracy of coal resource estimates and, ultimately, on the investment risk. The thickness of a coal seam determines the total resource tonnage whilst the quality parameters (calorific value, ash content and total sulphur content) determine the coal price.

There are several ways in which geostatistics can improve coal seam resource estimation (Larkin, 2009; Olea et al., 2011):

- Estimation of coal seam thickness and overburden volumes.
- Estimation of coal seam quality parameters.
- Provision of confidence limits on these estimations.
- Optimisation of drilling: determining where to place additional drill holes and determining the minimum drill hole spacing for the classification of resources as measured, indicated or inferred.
- Simulation of mining operations to achieve a particular objective such as minimising the variability of the quality characteristics of mined or stock-piled product so as, for example, to meet sales contract specifications.

Although kriging has been used extensively to address most of these problems (Noppé, 1994; Demirel et al., 2000; Watson et al., 2001) it has two important drawbacks. Firstly, although kriging is distribution-free and provides the estimation variance as a measure of the uncertainty of the estimates, a distribution must be assumed to provide confidence limits for the estimates; in addition, the estimates are spatially correlated as are the measures of uncertainty. The usual approach is to assume a Gaussian (Normal) distribution because of its simplicity but other more suitable alternatives can be found at the expense of increased computing time. Secondly, kriging provides estimated values of the coal seam variables that have less variability than the real values, i.e., the estimated values are smoother than reality.

Geostatistical simulation is a well-established technique (e.g., Journel and Huijbregts, 1978; Remy et al., 2009) that can be used to generate realisations that reproduce the spatial variability of coal seam parameters. In applications, a set of simulated realisations is used to assess the impact of the spatial uncertainty of the variables of interest (e.g., thickness, coal quality variables). The simulation, however, is based on a semi-variogram model that is estimated from the relatively sparse, available data and thus the model itself is uncertain. We quantify the uncertainty of the semi-variogram model by assessing the uncertainty of the semi-variogram model parameters and considering a set of likely model parameters. This quantification provides a means

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of propagating the uncertainty in the semi-variogram model parameters into the uncertainty of the estimated coal seam variables (Pardo-Igúzquiza and Chica-Olmo, 2008).

The consequence of many depositional mechanisms, such as alluvial fans that have in-filled basins, is a directional thickening or thinning of a seam; tectonic mechanisms, such as rifting or differential subsidence, can have similar consequences. In geostatistical terminology the systematic increase or decrease of seam thickness in one or more directions is termed a drift or trend, which must be accounted for in estimation and simulation. The two common ways of accommodating drift in estimation and simulation are explicit modelling and the use of intrinsic random functions (Wackernagel, 2003) although, as we expound in the following section, there is a more parsimonious solution.

2. Methodology

The estimation of coal tonnage and coal quality is based on the available data, which, at the resource estimation stage, is usually limited to exploration and feasibility drill holes. The significant advantage of geostatistical methods over other methods (e.g., polygonal, inverse distance) of estimating resources and reserves is that they are probabilistic. The outcome of a probabilistic process (estimation or simulation) is a random field of spatially variable values. Two decisions must be taken in any applied geostatistical study:

- whether the variable of interest (e.g., thickness) can be considered stationary (intrinsic or second-order) (Myers, 1989) or there is a trend that must be modelled; and
- how a semi-variogram model can be estimated from the data.

With respect to the first question, mean coal seam thickness often exhibits a trend that reflects the geological origin of the coal from sediment deposition within a basin or sub-basins and the effects of differential subsidence, or other mechanisms, on those basins. In principle, a non-stationary model is the most appropriate for estimation in such cases. However, coal resources are usually estimated from data from exploration and feasibility drill holes, on a more or less regular grid, that provide good coverage of the area of interest; in such cases the estimation of resources is similar to an interpolation problem. In this situation, Journel and Rossi (1989) have shown that a stationary model within a local search window gives very similar results to those obtained from a non-stationary model in which the drift is explicitly included. The presence of sufficient data, regularly distributed over the area of interest, has the double effect of conditioning and screening. Conditioning implies that, even if a stationary model is used within a local search window, the global drift is implicitly introduced into the final fields (whether estimated or simulated). Screening implies that, at an unsampled location, the closest neighbours are the most informative and influential values, in both kriging and simulation, and they screen the effect of neighbours that are further away from the location at which a value is to be estimated or simulated.

With respect to the second question, visual fitting is very common in mining applications largely because a competent resource estimator can take into account his/her own knowledge of the geology and structure of the deposit to interpret the behaviour of the semi-variogram (e.g., nugget effect, anisotropy, zonal effect). However, when a semi-variogram model is estimated from a small number of data, the parameters of the model have an associated uncertainty that must be assessed and included in estimation or simulation. Thus a parametric approach, such as maximum likelihood estimation, is more appropriate because it provides the uncertainty of the parameters of the semi-variogram model that is fitted to the calculated values (Pardo-Igúzquiza, 1998). Nevertheless, there are other alternatives for modelling the uncertainty of semi-variogram parameters such as bootstrap methods (Olea and Pardo-Igúzquiza, 2011) and Bayesian procedures (Kitanidis, 1986).

Given a set of n data $\mathbf{z}$, the semi-variogram parameter estimates are obtained as the values that minimise the negative log-likelihood function (ℓ) given by (Kitanidis, 1983):

$$\ell\left(\boldsymbol{\theta},\sigma^2;\hat{\beta},\mathbf{z}\right)=\frac{n}{2}\ln(2\pi)+\frac{n}{2}\ln\left(\sigma^2\right)+\frac{1}{2}\ln|\mathbf{Q}| + \frac{1}{2\sigma^2}\left[(\mathbf{z}-\boldsymbol{\beta})'Q^{-1}(\mathbf{z}-\boldsymbol{\beta})\right] \quad (1)$$

where, $\boldsymbol{\theta}'=(r_0,a)$ is the vector of correlogram parameters: the nugget/variance ratio and the range respectively. σ^2 is the variance parameter. $\mathbf{Q}=\mathbf{Q}(r_0,a)$ is the $n\times n$ correlation matrix, which depends on r_0 and a.

$\hat{\beta}$ is the estimated global mean and $\mathbf{z}'=(z(\mathbf{u}_1),z(\mathbf{u}_2),\ldots,z(\mathbf{u}_n))$ is the $1\times n$ vector of n spatial data with the general element $z(\mathbf{u}_i)$ being the ith data value at the spatial location $\mathbf{u}_i=\{x_i,y_i\}$.$\boldsymbol{\beta}'=\left(\hat{\beta},\hat{\beta},\ldots,\hat{\beta}\right)$ is the $1\times n$ vector with all elements equal to the global mean, which is estimated as:

$$\hat{\beta}=\left(\mathbf{1}'\mathbf{Q}^{-1}\mathbf{1}\right)\mathbf{1}'\mathbf{Q}^{-1}\mathbf{z} \quad (2)$$

$\mathbf{1}'=(1,1,\ldots,1)$ is a $1\times n$ vector of ones.

The maximum likelihood estimates, $\hat{\boldsymbol{\theta}}^*=\left\{\hat{\boldsymbol{\theta}},\hat{\sigma}^2\right\}$, of the semi-variogram parameters, $\boldsymbol{\theta}$ and σ^2, are the values that minimise the NLLF $\ell(;)$ in Eq. (1). Software for obtaining the maximum likelihood estimates may be found in Pardo-Igúzquiza (1997) and Diggle and Ribeiro (2007) among others.

One way of providing confidence regions (that is, sets of parameter values that have a given probability of being the true parameter values) is by using likelihood regions (sets of parameters which the NLLF is smaller than a given threshold) and the likelihood ratio statistic (Kalbfleisch, 1979):

$$D(\boldsymbol{\theta}^*)=-2\ln\frac{\text{likelihood}(\boldsymbol{\theta}^*)}{\text{likelihood}\left(\hat{\boldsymbol{\theta}}^*\right)} \quad (3)$$

That is,

$$D(\boldsymbol{\theta}^*)=2\left[\ell(\boldsymbol{\theta}^*)-\ell\left(\hat{\boldsymbol{\theta}}^*\right)\right] \quad (4)$$

where $D(\boldsymbol{\theta}^*)$ is the likelihood ratio statistic; $\ell(\boldsymbol{\theta}^*)=-\ln\{\text{likelihood}(\boldsymbol{\theta}^*)\}$ is the NLLF value for a set of arbitrary covariance parameters $\boldsymbol{\theta}^*$; $\ell\left(\hat{\boldsymbol{\theta}}\right)$ is the NLLF value for the set of ML covariance parameter estimates $\hat{\boldsymbol{\theta}}$.

It can be shown that for large n, $D(\boldsymbol{\theta}^*)$ is approximately chi-square distributed with p degrees of freedom (McCullagh and Nelder, 1989):

$$D(\boldsymbol{\theta}^*)\approx\chi^2_p \quad (5)$$

where the symbol $\approx$ denotes "distributed as", χ^2_p is the chi-square distribution with p degrees of freedom where p is the number of covariance parameters. In our case of Gaussian likelihood, $D(\theta^*)$ will have a chi-square distribution without any further assumption. Even for more complicated cases the distribution is asymptotically chi-square (Wilks, 1953) or a variation of chi-square (Chernoff, 1953).

Hence, for example, an approximate 75% confidence interval for the three parameters ($p=3$) (nugget variance, variance and range) is defined by the set of parameters with NLLF satisfying the criterion:

$$\ell(\boldsymbol{\theta}^*)<\ell\left(\hat{\boldsymbol{\theta}}^*\right)+\frac{1}{2}(4.108) \quad (6)$$

An application is presented in the following case study.