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Application and effect of buoyancy on sucker rod string dynamics

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ABSTRACT

The dynamic behavior of a sucker rod string is usually governed by a wave equation, which is used for a sucker rod pumping system design or diagnosing downhole working conditions. The wave equation is deduced by analyzing the force and moment applied on the rod string. Buoyancy is one of the forces distributed along the rod string in deviated wells and coupled with side force between the rod string and the tubing. Buoyancy also affects the downhole load of the subsurface pump. However, there is little knowledge concerning how buoyancy affects the dynamic behavior of the sucker rod string.

In this paper, three buoyancy handling methods are analyzed. These methods include the conventionally used buoyancy factor method (BFM), and the end force method (EFM) which ignores the fluid force on the cylindrical surface of the rod string. A new direct calculation method (DCM) that directly calculates the fluid force acting on the curved cylindrical surface of the rod element is presented. The wave equations using three different buoyancy handling methods are deduced, and their different subsurface dynamometer cards as boundary conditions are discussed. The solutions for three different wave equations are either analytically or numerically compared. The results indicate that the solutions of BFM and DCM are equivalent and differ by a fictitious term. The solutions of DCM and EFM have subtle distinction due to relative small distributed fluid force on the cylindrical surface. This study characterizes the effect of buoyancy, as well as supporting the unproved reasonableness of BFM and EFM on the sucker rod string dynamics. The presented DCM can also contribute to more precise prediction or diagnosis of downhole working conditions.

1. Introduction

Sucker rod pumping systems are extensively used for artificial lift in oil wells. Thus, it is of great importance to have an accurate understanding of sucker rod pumping process. The rod string dynamics is used to reveal the dynamic behavior of sucker rod pumping systems, which is critical to produce possible maximum oil with minimum production cost.

The sucker rod string dynamics is primarily restricted by the rod elasticity, the friction force along rod string and the boundary condition of the rod string. Because the rod string is immersed in the well fluids, it is acted on by a buoyant force that influences the friction force as well as the boundary condition of the rod string. The buoyancy is the resultant force of the distributed fluid force on the cylindrical surface and the concentrated force on the bottom of the rod string. The fluid force on the cylindrical surface for the vertical wells is zero, whereas it is perpendicular to the tangent of the rod string and coupled with the side force in the same direction for deviated wells. The concentrated fluid force on the bottom is coupled with the subsurface pump's load. Thus, it is of practical significance to investigate the effect of buoyancy on the sucker rod string dynamics.

A simple condition exists in vertical wells because both the side force and fluid force on the cylindrical surface of the rod string are zero. The buoyancy of the rod string is equal to the concentrated fluid force on the bottom end of the rod string. Commonly, buoyancy is treated with the total weight of rod string in air by using the rod weight in the fluid (Takács, 2003). Via this treatment, the pump's load in ideal condition is zero during the downstroke and of the full plunger area fluid column weight during the upstroke. (Gibbs, 1963; Gibbs and Neely, 1966) first presented the wave equation, a second-order partial differential equation, to describe the rod string dynamics, and used the rod weight in the fluid to handle the buoyancy. Regarding other improvements of the one-dimensional wave equation considering liquid and tubing vibration (Doty and Schmidt, 1983; Wang et al., 1992; Lollback et al., 1997; Luan et al., 2012), the buoyancy is not considered on the motion equation of the rod element to yield the wave equation. The buoyancy is only applied on the downhole boundary condition.

The dynamic behavior of the sucker rod string in a deviated well with a wellbore trajectory in three-dimensional space is more

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Nomenclature

A_p	area of the plunger (m^2)
A_r	cross-section area of the rod string (m^2)
b	binormal unit vector
$\boldsymbol{d}_0$	steepest ascent vector
E_r	Young's modulus of the rod string (Pa)
F	true force on cross section of the rod element (N)
$\bar{F}$	effective force on cross section of the rod element (N)
F_{axial}	axial force (N)
F_f	fluid force per unit length (N/m)
f_b	binormal component of the fluid force per unit length (N/m)
f_n	normal component of the fluid force per unit length (N/m)
f_λ	Columbia friction per unit length (N/m)
f_μ	viscous friction per unit length (N/m)
g	gravity acceleration (m/s^2)
H	vertical depth (m)
I	moment of inertia (m^4)
$\boldsymbol{i}$	northern unit vector
$\boldsymbol{j}$	eastern unit vector
$\boldsymbol{k}$	vertical unit vector
k	curvature (m^{-1})
M	moment on the cross section of the rod element ($N \cdot m$)
N	side force per unit length between the rod string and the tubing (N/m)
N_b	binormal component of the side force per unit length (N/m)
N_n	normal component of the side force per unit length (N/m)
$\boldsymbol{n}$	normal unit vector
P_p	load on the plunger of the subsurface pump (N)
p	pressure in the tubing (Pa)
p_a	pressure on the stripe area (Pa)
p_i	intake pressure of the pump (Pa)
p_o	discharge pressure of the pump (Pa)
p_p	pressure at pump chamber (Pa)
q_r	rod weight per unit length in air (N/m)
$\bar{q}_r$	rod weight per unit length in the fluid (N/m)
s	well measure depth (m)
Δs	length of the rod element (m)
T	torsion (m^{-1})
t	time (s)
u	displacement of the rod element (m)

Greek symbols

α	inclination angle (rad)
β	buoyancy factor (dimensionless)
λ	Columbia friction coefficient (dimensionless)
ν	viscous coefficient ($N \cdot s/m^2$)
ρ_l	density of the fluid (Kg/m^3)
ρ_r	density of the rod string (Kg/m^3)
ϕ	azimuth angle (rad)

Subscripts

bf	buoyancy factor method
dc	direct calculation method

complicated. The key element to the model's exactness is the friction law on the rod string (Gibbs, 1992; DaCunha and Gibbs, 2009). The Columbia friction is directly related with the side force, and the side force is coupled with the fluid force on the cylindrical surface. In the literature to date, the fluid force on the cylindrical surface of the rod element, which is also referred to as the buoyancy of the rod element in this paper, is either ignored (Lukasiewicz, 1991; Gibbs, 1992; Xu, 1994; Mo and Xu, 2000; Liu et al., 2004; Araújo et al., 2015), or equal with the weight of displaced fluid according to Archimedes' Law (Li et al., 1995; Vasserman and Shardakov, 2003; Wang, 2010). Thus, the application of buoyancy on rod string dynamics is the same as that of vertical wells. To better reveal the dynamic behavior of rod string in deviated wells, it is important to clarify the application of buoyancy and its effect on the rod string.

The buoyancy is also met in tubular buckling analysis. It is customary to use the buoyancy factor to address the buoyant effect. Lubinski (1950, 1951, 1975) used an equivalent transformation from the cylindrical fluid force on the tubular to the buoyant force of the tubular element and an additional fictitious axial force. Aadnoy and Kaarstad (2006) noted that it was more interesting to apply the buoyancy factor method to obtain the effective force, which led to tubular failure rather than the true axial force. Aadnoy et al. (2010) also used the effective string weight to determine the wellbore friction.

As the above review of the literature shows, although a number of authors had studied the dynamic behavior of the rod string using the effective rod weight in the fluid or ignoring the cylindrical fluid force and found the results were acceptable in practice, the discussion on the application and effect of buoyancy on the rod string dynamics is still obscure and has not appeared in publications. To characterize the mechanism of action of buoyancy and improve the accuracy of the dynamics model, in this paper, the buoyant force on the rod element is directly calculated and later used in the wave equation to compare with two other buoyancy handling methods.

2. Application of buoyancy on the rod element

The rod string of a sucker rod pumping system in a deviated well is curved and immersed in the oil fluid (Fig. 1). A rod element is exposed to fluid pressure on the cylindrical surface. The total fluid force on the element's surface can be manipulated in three ways. As is mentioned in the introduction, one way considers the rod weight in the fluid, which is referred to as the buoyancy factor method (BFM). Another way ignores the cylindrical fluid force on the rod element and only considers the bottom end fluid force on the rod string, which is referred to as the end force method (EFM). Finally, a new direct calculation method (DCM) to analyze the fluid force on rod's cylindrical surface is presented in this paper, which directly calculates the fluid force on cylindrical surface of the rod element. Because the buoyancy or fluid force is distributed along the rod string, it is discussed per unit length in this paper.

2.1. Buoyancy factor method

The buoyancy factor method was first presented by Lubinski (1950, 1951) in dealing with tubular buckling. In that method, the cylindrical pressure force on the element is transformed to the element buoyancy and the axial forces at both element ends, as shown in Fig. 2. First, each end face of the rod element is assumed