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Black-oil minimal fluid state parametrization for constrained reservoir control optimization

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ABSTRACT

We propose to solve a black-oil reservoir optimal control problem with the Direct Multiple Shooting Method (MS). MS allows for parallelization of the simulation time and the handling of output constraints. However, it requires continuity constraints on state variables to couple simulation intervals. The black-oil fluid model, considering volatile oil or wet gas, requires a change of primary variables for simulation. This is a consequence of the absence of a fluid phase due to dissolution or vaporization. Therefore, reservoir simulators parametrize the states with an augmented vector and select primary variables accordingly. However, the augmented state vector and the corresponding change of primary variables are not suitable for the application of MS because the optimization problem formulation must change according to the change of variables. Thus, we propose a minimal state-space variable representation that prevents this shortcoming. We show that there is a bijective mapping between the proposed state-space representation and the augmented state-space. The minimal representation is used for optimization and the augmented representation for simulation, thereby keeping the simulator implementation unchanged. Therefore, the proposed solution is not invasive. Finally, the application of the method is exemplified with benchmark cases involving live oil or wet gas. Both examples emphasize the requirement of output constraints which are efficiently dealt with the MS method.

1. Introduction

The Direct Multiple Shooting Method (MS) is an effective technique to deal with output constraints in control optimization of two-phase reservoirs (Codas et al., 2015). However, the extension of this method to three-phase black-oil models is not trivial due to the reservoir grid-block state-space representation of the fluid saturation condition. This work extends the MS method in (Codas et al., 2015) to miscible black-oil fluid models including live oil and wet gas. Compared to immiscible models, the miscible black-oil model requires additional analysis of the fluid state during simulation to determine the fluid flow conditions appropriately.

Black-oil models are convenient due to their computational simplicity and their capability to approximate compositional models (Fevang et al., 2000). Black-oil models can be seen as a special case of compositional models with three components, water, oil, and gas, associated to three reservoir phases, aqueous, liquid and vapor, respectively. A component may be seen as an indivisible set of molecules which are transported within a fluid phase. A phase is a mixture of components, a homogeneous part of the fluid which is separated of other phases by a boundary surface. The oil and gas chemical components are typically defined as the composition of the liquid phase and vapor phase at standard conditions, respectively. Here, the water component is treated as an immiscible component found exclusively in the aqueous phase. Moreover, oil and gas are the main components of the liquid and vapor phases. However, oil and gas components may exist in both the liquid and vapor phases in the reservoir pressure and temperature conditions. Wet gas models consider a fraction of the oil components vaporized in the vapor phase, whereas live oil models consider a fraction of gas components dissolved in the liquid phase.

Depending on the components properties and reservoir conditions, a component may exist in the fluid mixture while its associated phase may be absent (Mattax and Dalton, 1990; Chen et al., 2006). For instance, the gas and oil components may be completely dissolved in the liquid phase and in this case no vapor phase exists. In live oil models, there is a maximum amount of gas

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components that can be dissolved in the liquid phase at a given pressure and temperature condition. If the gas components found in the fluid do not reach this maximum amount then the fluid is regarded as under-saturated. In under-saturated conditions, no vapor phase is present. The vapor phase appears if and only if the liquid phase gets saturated of gas, for instance, as a consequence of a drop of pressure below the bubble point pressure. Analogously, wet gas models have an under-saturated and saturated state depending on the fluid conditions, and the existence of the liquid phase depends on the saturation condition.

Compared to compositional simulators, black-oil simulators dispense with the equations of state needed to define component mass fractions. During simulation, the fluid state is monitored and the set of equations describing the fluid flow is switched when a phase appears or disappears (Chen et al., 2006). Thus, any optimization procedure handling this simulator must be capable to determine when the transition occurs and switch the set of equations and primary variables accordingly. Therefore, the optimizer inherits the simulator complexity. Further, since the state representation is discontinuous, MS optimizers face an additional complexity to estimate predicted states.

The application of optimal control to improve the economical return of oil reservoirs described by three-phase black-oil models is not new as Zakirov et al. (1996) applied the Conjugate Gradient method to solve this problem. A real field example considering a fluid model with gas soluble in the oil phase was optimized by Davidson and Beckner (2003) using the Sequential Quadratic Programming method. Recently, Krogstad et al. (2014) optimized a reservoir model including live oil with a line-search method and a heuristic control-switching method to handle output constraints. However, in the previous works the state variables are not explicitly available in the optimization method as in MS, therefore the specific representation of the state variables does not impose any problem. Key advantages that come with the explicit representation of the states are simulation parallelization opportunities and easy output-constraint handling (Codas et al., 2015).

This work aims to develop a MS formulation for control optimization of oil reservoirs modeled with the black-oil model including gas. In Section 2.1 we describe the reservoir model and Appendix A presents a simplified procedure to solve it. Then, in Section 3 we develop a minimal state parametrization to represent the fluid state that is suitable for a MS optimal control problem formulation. In the following section we demonstrate the applicability of this new formulation to simple test cases. Finally, this work ends with a discussion of the results and a brief conclusion in Sections 5 and 6, respectively.

2. Reservoir model

This works aims to develop a MS formulation which adapts tightly to black-oil reservoir simulators containing miscible hydrocarbons. This section briefly presents the equations being solved in such reservoir models. The solution procedure described in Appendix A is taken from the Matlab Reservoir Simulation Toolbox (MRST) (Lie et al., 2011; Krogstad et al., 2015) which is later used in our test cases.

2.1. The miscible black-oil flow in porous media

The mass conservation principle, the capillary pressure phenomenon, the Darcy law and an empirical modeling of components miscibility given by the black-oil model lead to the differential equations describing three-phase flow in porous media (Chen et al., 2006, p. 283):

$$\frac{\partial}{\partial t}\left(\frac{\phi S_a}{B_a}\right) = -\nabla\cdot\left(\mathbf{T}_a\nabla\mathbf{\Phi}_a\right) + \frac{q_a}{B_a}, \tag{1a}$$

$$\frac{\partial}{\partial t}\left[\phi\left(\frac{S_l}{B_l} + \frac{R_v S_v}{B_v}\right)\right] = -\nabla\cdot\left(\mathbf{T}_l\nabla\mathbf{\Phi}_l + R_v\mathbf{T}_v\nabla\mathbf{\Phi}_v\right) + \frac{q_l}{B_l} + \frac{q_v R_v}{B_v}, \tag{1b}$$

$$\frac{\partial}{\partial t}\left[\phi\left(\frac{S_v}{B_v} + \frac{R_l S_l}{B_l}\right)\right] = -\nabla\cdot\left(\mathbf{T}_v\nabla\mathbf{\Phi}_v + R_l\mathbf{T}_l\nabla\mathbf{\Phi}_l\right) + \frac{q_v}{B_v} + \frac{q_l R_l}{B_l}, \tag{1c}$$

$$S_a + S_l + S_v = 1, \tag{1d}$$

$$p_{cla} = p_l - p_a, \quad p_{cvl} = p_v - p_l, \tag{1e}$$

$$\mathbf{\Phi}_\alpha = p_\alpha - \rho_\alpha\|\mathbf{g}\|z, \quad \mathbf{T}_\alpha = \lambda_\alpha\mathbf{k} = \frac{k_{r\alpha}}{\mu_\alpha B_\alpha}\mathbf{k}, \quad \alpha\in\{a, l, v\}; \tag{1f}$$

The nomenclature for (1) is presented in Table 1.

The gas solubility and the oil volatility determine the fluid miscibility and its saturation state. The gas solubility and the oil volatility range from zero (for immiscible fluids) to a maximum value given by a saturation function, i.e., $R_v \le R_v^{\max}$ and $R_l \le R_l^{\max}$. Furthermore, it is assumed that a phase can exist only if the reciprocal phase is saturated, i.e., $S_v > 0 \to R_l = R_l^{\max}$ and $R_l < R_l^{\max} \to S_v = 0$ ($S_l > 0 \to R_v = R_v^{\max}$ and $R_v < R_v^{\max} \to S_l = 0$).

Table 1
Nomenclature.

Variable	Description
t	Time
$\{a, l, v\}$	Set of phases (aqua, liquid and vapor)
$\{W, O, G\}$	Set of components (water, oil and gas)
ϕ	Rock porosity
S_α	Saturation of the phase α
B_α	Phase α's formation volume factor
$q_{\beta s}$	Standard volumetric flow of component β injected or produced through the wells
$k_{r\alpha}$	Phase α's relative permeability
μ_α	Phase α's viscosity
$\mathbf{k}$	Rock absolute permeability
p_α	Phase α's absolute pressure
ρ_α	Phase α's density
$\|\mathbf{g}\|$	Gravity absolute value
z	Height (increases in the same direction as the gravity)
p_{cla}	Liquid-aqueous capillary pressure
p_{cvl}	Vapor–liquid capillary pressure
R_v	Oil volatility in the vapor phase
R_l	Gas solubility in the liquid phase
$\mathbf{x}$	Space coordinates
$\mathcal{W}$	Set of wells
$\mathcal{M}^w$	Set of perforations of well w
$W^{\mathrm{I}}_{w,m}$	Well index related to the perforation m of well w located at $\mathbf{x}^{w,m}$
p^w_{bh}	Bottom hole pressure of well w
N_P	Number of phases
N_C	Number of components
χ	Five dimensional grid-block state variable $(p_l, S_a, S_v, R_l, R_v)$
γ	Three dimensional grid-block state variable (p_l, S_a, r_H)
Γ	Transformation taking χ to γ
(r_O, r_G, r_H)	See Eqs. (3)
$\mathcal{S}$	Saturation state, see Table A1
ζ	Simulation primary variable, see Table A1
(w_O, w_G)	Volume fraction of oil and gas at standard conditions