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Rate forecasting during boundary-dominated multiphase flow: The rescaled exponential model

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ABSTRACT

Well performance forecasting is an important analytical technique used for field development to guide economic decisions during the life of a reservoir. For the case of dry and liquid-rich gas wells, traditional well performance models are developed based on solving the resultant highly nonlinear gas flow equations via pseudo-pressure and pseudo-time linearization. In this study, we provide a straightforward, density-based alternative to traditional models. We show, as done previously for the case of dry gas wells (Ayala and Zhang, 2013; Zhang and Ayala, 2014a), that a rescaled exponential model is a rigorous decline solution that can be extended to liquid-rich gas wells producing under constant bottom-hole-flowing-pressure (BHP) during boundary-dominated flow (BDF) and multiphase conditions. The proposed multiphase rescaled-exponential model is derived analytically from governing multiphase flow equations; comparisons between numerically simulated results and proposed analytical model for a variety of combinations of reservoir and fluid properties demonstrate that the proposed rescaled-exponential model is a valid and reliable forecast model for constant-BHP liquid-rich gas wells under BDF.

1. Introduction

Modern well performance analysis techniques can trace their roots back to the work of Fetkovich (1980), who presented a group of well performance analysis type-curves generated through a combination of analytical solutions and Arps (1945) empirical developments. For the case of gas systems, strongly pressure-dependent fluid properties introduce significant complexities and non-linearities into the analysis. For such cases, the concepts of pseudo-pressure (Al-Hussainy et al., 1966) and pseudo-time (Agarwal, 1979; Fraim and Wattenbarger, 1987) have entered the analysis in order to account for these non-linearities in the gas flow performance equations. Palacio and Blasingame (1993) applied them to their dry-gas, single-phase well performance analysis technique and introduced the material-balance pseudo-time to account for variable rate/pressure conditions during boundary dominated flow (BDF).

For the analysis of multiphase flow in liquid-rich gas or gas-condensate reservoirs, two-phase pseudo functions have also been developed to linearize the associated multiphase governing flow equations—in direct analogy to single-phase gas developments. O'Dell (1967) and Fussel (1973) used two-phase pseudo-pressure (m_{tp}) to predict gas condensate well performance through the gas rate deliverability equation. The two-phase pseudo-pressure, m_{tp}, is defined as:

$$m_{tp} = \int_{p_b}^{p} \left(\frac{k_{rg}}{\mu_g B_g} + R_s \frac{k_{ro}}{\mu_o B_o} \right) dp \tag{1}$$

where R_s is the solution gas–oil ratio, μ *and* B are the fluid viscosity and formation volume factors, respectively and k_r refers to relative permeability of oil and gas, denoted by the subscript 'o' and 'g' respectively. Jones and Raghavan (1988) implemented the two-phase pseudo-pressure Eq. (1) in the development of well test analysis methods for gas-condensate reservoirs. Modifying the equation originally proposed by Evinger and Muskat (1942) for solution-gas-drive oil wells, Fevang and Whitson (1996) presented the pseudo-steady-state equation for gas-condensate wells using pseudo-pressures calculated from a pressure-saturation relationship obtained from the producing gas-oil-ratio (GOR). Honoring such calculation of two-phase pseudo-pressure, Sureshjani and Gerami (2011) proposed the two-phase pseudo-time, $t_{a,tp}$, as a function of gas and oil saturations, S_g and S_o, for gas-condensate reservoirs under variable rate/pressure conditions in a form similar to that used for dry gas reservoirs:

$$t_{a,tp}(t) = \frac{1}{m_{tp}(p_i)} \int_0^t \frac{dt}{\frac{\partial}{\partial m_{tp}} \left(\frac{S_g}{B_g} + R_s \frac{S_o}{B_o} \right)} \tag{2}$$

An analogy of the governing equation for gas-condensate flow with that of single phase liquid flow may be established when

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Nomenclature

A	reservoir area, ft^2
$b_{D,PSS}$	pseudosteady state component
B_g	gas formation volume factor, RB/SCF
B_o	oil formation volume factor, RB/STB
$\bar{c}_g^*$	equivalent gas compressibility, 1/psi
$\bar{\mu}_g^*$	equivalent gas compressibility at average reservoir condition, 1/psi
D_i^e	initial decline coefficient for dry gas rescaled exponential model under full potential drawdown, 1/day
$D_{tp,i}^e$	two-phase initial decline coefficient for rescaled exponential model, lbm/SCF /day
G_i	original gas in place, SCF.
G_p	cumulative gas production, SCF
h	reservoir thickness, ft
k	absolute permeability, mD
k_{rg}	gas relative permeability
k_{ro}	oil relative permeability
m	pseudopressure, psi^2/cp
m_{tp}	two-phase pseudopressure psi /cp
$m_{tp,i}$	two-phase pseudopressure at initial condition, psi /cp
$m_{tp,wf}$	two-phase pseudopressure at wellbore condition, psi /cp
$m_{tp,re}$	two-phase pseudopressure at reservoir external boundary condition, psi /cp
$\bar{m}_{tp}$	two-phase pseudopressure at reservoir average condition, psi /cp
n_i	initial hydrocarbon moles in place, lbmol
p	pressure, psia
$\bar{p}$	average reservoir pressure, psia
p_i	initial reservoir pressure, psia
p_{dew}	dewpoint pressure, psia
p_{sc}	pressure at standard condition, psia
p_{wf}	well bottom-hole pressure, psia
q_{gi}^e	initial decline rate for dry gas re-scaled exponential model under full potential drawdown, SCF/D
$q_{gtp,i}^e$	two-phase initial decline moles of gas component for re-scaled exponential model, lbm/day
q_{gsc}	surface gas rate, SCF/D
q_{osc}	surface oil rate, STB/D
q_{Dd}	dimensionless rate used in Arps' decline model
R	universal gas constant
R_s	solution gas-oil-ratio, SCF/STB
R_p	produced gas-oil-ratio, SCF/STB
R_v	volatile oil-gas-ratio, STB/SCF
r	radial distance, ft
r_w	wellbore radius, ft
r_e	reservoir external radius, ft
r_ρ	single-phase gas density drawdown ratio
$r_{\rho,tp}^*$	two-phase density drawdown ratio
S_g	gas saturation
S_o	oil saturation
S_{wc}	connate water saturation
T	reservoir temperature, °F or °R
T_{sc}	temperature at standard condition, °F or °R
t	time, day
t_{DAd}	dimensionless time used in Arps' decline model
$t_{a,tp}$	two-phase pseudotime, day
$t_{acr,tp}$	two-phase material-balance pseudo-time, day
v_g	gas phase velocity, (L/t)
v_o	oil phase velocity, (L/t)
V_p	reservoir pore volume, ft^3
z_{tp}	two-phase compressibility factor

Greek

$\bar{\lambda}$	depletion-driven viscosity-compressibility ratio for dry gases, dimensionless
$\bar{\lambda}_{tp}$	depletion-driven viscosity-compressibility ratio of equivalent surface gas component during two-phase flow, dimensionless
$\bar{\beta}$	time-averaged of the dry gas depletion parameter $\bar{\lambda}$, dimensionless
$\bar{\beta}_{tp}$	depletion-driven time-averaged evolution of viscosity-compressibility ratio of equivalent gas component in two-phase flow $\bar{\lambda}_{tp}$, dimensionless
ρ_g^*	equivalent gas component molar density, lbmol/RB
$\rho_{g,i}^*$	equivalent gas component molar density at initial reservoir condition, lbmol/RB
$\rho_{g,wf}^*$	equivalent gas component molar density at wellbore condition, lbmol/RB
$\bar{\rho}_g^*$	equivalent gas component molar density at average reservoir condition, lbmol/RB
ρ_{sc}^g	equivalent gas component molar density at standard condition, lbmol/SCF
ϕ	porosity
μ_g	gas viscosity, cp
μ_o	oil viscosity, cp
μ_g^*	equivalent gas viscosity, cp
	μ

properly defined two-phase pseudo-functions—such as pseudo-pressure and pseudo-time are used. Besides the rigorously-derived two-phase pseudo-functions, other attempts have been made to simplify the calculation of two-phase pseudo-functions in production data analysis (PDA). For example, Sureshjani et al. (2014) proposed an empirically-defined two-phase pseudo-pressure and pseudo-time for gas condensate well by neglecting the condensate mobility term $\frac{R_s k_{ro}}{\mu_o B_o}$ in both equations (Eqs. (1) and 2). Arabloo et al. (2014) used two-phase Z-factor (Z_{tp}) (Hagoort, 1988) in their definition of pseudo-functions, which results in a two-phase pseudo-pressure and pseudo-time form, analogous to the dry-gas case.

Recently, Ayala and Zhang (2013) and Zhang and Ayala (2014a, 2014b) proved that dry gas well decline under constant bottom-hole pressure can be successfully captured in terms of a re-scaled exponential model:

$$q_{gsc} = q_{gi}^e \cdot r_\rho \cdot \bar{\lambda} \cdot \exp\left(-D_i^e \cdot \bar{\beta} t\right) \tag{3}$$

where $\bar{\lambda}$ and $\bar{\beta}$ are dimensionless rescaling parameters quantifying the effect of non-linearities (i.e., gas viscosity-compressibility dependencies with pressure) in the performance equation, r_ρ is the density drawdown ratio, q_{gi}^e is the initial decline rate, and D_i^e is the initial decline coefficient. The authors showed that this re-scaled exponential model can be rigorously and analytically derived for both constant and variable BHP conditions for single-phase flow. This work extends the single-phase model to multi-phase flow environments while attempting to maintain their simplicity and analogy. This is made possible by the use of a density-based material balance approach for liquid-rich gases (Zhang and Ayala,