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An improved TSVD-based Levenberg–Marquardt algorithm for history matching and comparison with Gauss–Newton

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ABSTRACT

Parameterization based on truncated singular value decomposition (TSVD) of the dimensionless sensitivity matrix has been shown to be an efficient approach for history matching. With TSVD parameterization, the search direction is computed as a linear combination of a few principal right singular vectors. As the sensitivity matrix is not explicitly computed, this parameterization is appropriate for large-scale history-matching problems. Moreover, previous work presented theoretical evidence that TSVD of the dimensionless sensitivity matrix provides the optimal parameterization in terms of uncertainty reduction. TSVD has been used in the randomized maximum likelihood (RML) framework to generate multiple conditional realizations of reservoir models. In this work, we investigate the effect of TSVD in the search direction obtained by the application of the Gauss–Newton and the Levenberg–Marquardt (LM) methods. In particular, we show that the TSVD-based LM algorithm converges to appropriate estimates because it gradually resolves the important features of the true model. We also introduce an improved implementation of a TSVD-based LM algorithm for generating multiple realizations of reservoir models conditioned to production data. Our experiments indicate that the computational cost of the new implementation is on the order of 2/3 of the cost of the previous implementation.

1. Introduction

There is always uncertainty in the simulation models of petroleum reservoirs. Bayesian statistics provides a mathematical framework in which uncertainty can be described in terms of a probability density function (pdf) (Tarantola, 2005). The problem of describing uncertainty can then be replaced by a conceptually simpler problem of sampling a posterior pdf. Sampling the posterior pdf is equivalent to generating plausible realizations of the reservoir model. However, even the sampling problem can be quite complex and computationally expensive for reservoir applications. Markov chain Monte Carlo (MCMC) (Hastings, 1970; Tierney, 1994) provides a rigorous sampling method. However, MCMC is computationally too demanding and its direct application to reservoir problems seems not to be feasible with the current computational capabilities (Tjelmeland et al., 1994; Hegstad and Omre, 1999; Oliver et al., 1997), unless the size of the problem is small (Emerick and Reynolds, 2013a) or it is applied in combination with some approximation (proxy) for the likelihood (Emerick and Reynolds, 2012b).

The randomized maximum likelihood (RML) method was introduced by Oliver et al. (1996) as a practical approximation of MCMC for sampling in the context of reservoir applications. RML samples correctly when the data is linearly related to the model parameters. When the data–model relationship is nonlinear, RML is not guaranteed to sample correctly. However, several computational experiments (Liu and Oliver, 2003; Reynolds et al., 1999; Gao et al., 2006; Emerick and Reynolds, 2013a) have shown that RML performs an adequate sampling in the nonlinear case. For generating a realization of the posterior pdf with RML, an objective function should be minimized using an optimization algorithm. Hence, for generating N_e conditional reservoir models, N_e objective functions should be minimized. These minimizations are challenging optimization problems as history matching is usually ill-posed and the number of unknown model parameters can be very large. As a result, application of unsuited optimization algorithms may fail to provide an acceptable sampling of the posterior pdf.

Application of the Gauss–Newton (GN) and the Levenberg–Marquardt (LM) algorithms for solving the optimization problem involved in the history matching has been studied extensively (Reynolds et al., 1996; Abacioglu et al., 2001; Li et al., 2003; Rodrigues, 2006). Computing the GN or the LM search direction requires an explicit computation of the sensitivity matrix, which may not be feasible for large-scale problems, unless the number of

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data points or the number of model parameters is small. Hence, many researches have attempted to introduce efficient parameterization techniques to reduce the number of parameters when applying the GN or the LM algorithm.

Reynolds et al. (1996) used the subspace method in the GN algorithm to reduce the size of the matrix problem in each of the GN iterations for history matching problems. In the subspace method, the search direction vector is expanded as a linear combination of a few basis vectors. These vectors may be gradients of some sub-objective functions, e.g., gradient of data mismatch term and gradient of the model misfit part of the objective function. Abacioglu et al. (2001) followed the work of Reynolds et al. (1996) with a more detailed investigation. They applied both GN and LM algorithms with the subspace method. They suggested that instead of using a fixed number of subspace vectors, the number of subspace vectors can be gradually increased. They presented a theoretical argument that suggests the eigenvectors of the dimensionless matrix $L^{\mathrm{T}}G^{\mathrm{T}}C_{\mathrm{D}}^{-1}GL$ associated with the largest eigenvalues form an ideal basis for parameterization; here G is the sensitivity matrix, C_{D} is the covariance matrix for the measurement errors and L is the lower triangular matrix resulting from the Cholesky decomposition of the prior covariance matrix, C_{M}, which is written as $C_{\mathrm{M}} = LL^{\mathrm{T}}$. However, Abacioglu et al. (2001) did not use this approach in example problems. They mentioned that the computation of this set of vectors is probably too expensive to be practical. The parameterization they proposed as an ideal basis is actually equivalent to parameterizing the history matching search direction in terms of the right singular vectors of the dimensionless sensitivity matrix, $G_{\mathrm{D}} = C_{\mathrm{D}}^{-1/2}GC_{\mathrm{M}}^{1/2}$ (Oliver et al., 2008, Chapter 9), noticing that $G_{\mathrm{D}}^{\mathrm{T}}G_{\mathrm{D}} = L^{\mathrm{T}}G^{\mathrm{T}}C_{\mathrm{D}}^{-1}GL$, if one uses a Cholesky decomposition for computing $C_{\mathrm{M}}^{1/2}$, i.e., use $C_{\mathrm{M}}^{1/2} = L$. Eigenvectors of $L^{\mathrm{T}}G^{\mathrm{T}}C_{\mathrm{D}}^{-1}GL$ are the same as the right singular vectors of G_{D}, and the eigenvalues of $L^{\mathrm{T}}G^{\mathrm{T}}C_{\mathrm{D}}^{-1}GL$ are the squares of the singular values of G_{D} (Tavakoli and Reynolds, 2010). Abacioglu et al. (2001) observed that when applying the LM algorithm with parameterization, the initial rate of reduction in the objective function is largely independent of the number of subspace vectors.

Rodrigues (2006) applied a history matching procedure where the change in the vector of model parameters over a GN iteration is expressed as a linear combination of a few right singular vectors of G_{D}. To obtain the singular vectors corresponding to the largest singular values of G_{D}, he used the Lanczos algorithm (Golub and van Loan, 1989; Vogel and Wade, 1994). The main advantage of this procedure comes from the fact that Lanczos algorithm only requires the product of G times a vector and G^{T} times a vector. The first product can be efficiently computed with the gradient-simulator method (Anterion et al., 1989) and the second product can be computed with one adjoint solution (Li et al., 2003).

Tavakoli and Reynolds (2010) used the right singular vectors of G_{D} with the LM algorithm. Moreover, they presented a theoretical argument that suggests the principal right singular vectors of G_{D} form an optimal basis of parameterization, as eliminating those corresponding to smaller singular values has a negligible effect on the reduction of uncertainty obtained by conditioning a reservoir model to dynamic data. In their results, they obtained more reasonable permeability fields with the proposed TSVD parameterization than they obtained with application of a quasi-Newton method (LBFGS) (Nocedal, 1980). The effect of the TSVD parameterization is twofold: first, it makes the application of GN and LM methods computationally feasible for large-scale models; second, the parameterization introduces additional regularization which alleviates the ill-posedness of the inverse problem. Later, Dickstein et al. (2010) used a TSVD parameterization in the GN algorithm to condition the permeability fields to production and time-lapse seismic data. In their synthetic example, in the absence of seismic data, when they used a fixed number of singular triplets (25) with the GN algorithm, the resulting model was very rough, giving a poor representation of the reservoir, although the data matches were very good. However, they obtained good representations of the model starting with a few number of singular triplets and gradually increasing this number as iteration proceeds.

Li et al. (2003) applied the GN algorithm and a modified LM algorithm to generate the MAP estimate conditioned to pressure data for a simple 2D synthetic reservoir. In their results, the MAP estimate generated from the GN is very rough and far from the true model. They concluded that the modified LM algorithm is more reliable for obtaining good estimates of model parameters, because by using a very high value of the LM parameter at early iterations, the changes in model parameters are damped. Although they did not use a parameterization and they explicitly computed the sensitivity matrix in their implementation, their comparison of the results of the GN and the LM is in agreement with the results of Dickstein et al. (2010) and Tavakoli and Reynolds (2010).

Tavakoli and Reynolds (2011) used the TSVD parameterization in RML framework to simultaneously generate an ensemble of realizations of the reservoir model. They named the resulting algorithm SVD-EnRML (SVD-Ensemble-RML) and applied for history matching 2D permeability fields. Shirangi (2014) extended the SVD-EnRML algorithm for the simulation of porosity and permeability fields of 3D reservoir models, and also provided a modified algorithm to improve the computational efficiency. At each iteration of SVD-EnRML, a TSVD of G_{D} corresponding to a particular realizations (typically the MAP) is computed and used to update all realizations of the ensemble. Hence, compared to the original TSVD-based method (Tavakoli and Reynolds, 2010), the main source of efficiency of the SVD-EnRML is the fact that the same set of singular triplets is used to simultaneously minimize the objective functions of N_e realizations.

This paper has two main contributions. As the first contribution, we analyze the difference of the GN and the LM search directions when solving the history matching problem. Our analysis will provide more insight on why the LM algorithm provides appropriate solutions to the history matching problem (as observed by Tavakoli and Reynolds, 2010; Li et al., 2003; Abacioglu et al., 2001), while the GN algorithm may converge to poor/rough solutions (as observed by Dickstein et al., 2010; Li et al., 2003). We also provide a GN version of the SVD-EnRML algorithm that can be successfully applied to generate multiple realizations of the reservoir model. The second contribution of this work is that we introduce an inner loop procedure to improve computational performance of the LM-based SVD-EnRML. The improved algorithm is applied to 2D and 3D example problems.

The rest of this paper is outlined as follows. In the following section, we briefly explain the RML method for generating an approximate sampling of the posterior pdf. After that, we review the SVD-EnRML method. In the section after that, we compare GN and LM versions of the SVD-EnRML and investigate the effect of TSVD in the search direction. Then, we introduce the improved SVD-EnRML implementation, followed by two numerical examples. The last section of the paper summarizes our conclusions. We also present three appendix sections with throughout descriptions of the main algorithms discussed in this paper.

2. Sampling the posterior pdf with the randomized maximum likelihood method

Throughout, $\mathbf{m}$ denotes the N_m-dimensional vector of model parameters, which includes the unknown reservoir rock properties. When the prior geology is described in terms of the prior mean, $\mathbf{m}_{\mathrm{pr}}$, and the prior $N_m \times N_m$-dimensional covariance matrix, C_{M}, it is well-known that the maximum a posteriori estimate (MAP) (Oliver et al.,