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Numerical modelling of waterhammer pressure pulse propagation in sand reservoirs

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ABSTRACT

This paper presents a numerical model with a new approach for analyzing the propagation of pressure waves in porous media and investigates the dynamic response of sand in relation to the attributes of pore pressure pulses. There are various instances in which dynamic phenomena can have a significant impact on porous media in a reservoir. One notable example is the possible influence of waterhammer pressure pulsing on sand fluidization around injection wells in oil reservoirs following a hard wellbore shut-in, which can result in massive sand production. In some extreme cases, this phenomenon can even result in the loss of the wellbore. Nevertheless, the pore pressure wave propagation in porous media has often been neglected in modelling likely due to mathematical complexity.

The proposed model solves the momentum balance of fluid and solid coupled with the fluid mass balance equation in the prediction of dynamic fluid flow and mechanical deformation in porous media. The model is a two-dimensional, elasto-plastic, axisymmetric, single-phase and sequentially coupled model. The numerical model was validated against experimental data for a step wave in a shock tube and good agreement between model calculations and measured data has been obtained.

Two distinct waves have been observed as a result of a shock pore pressure wave. The first one is an undrained wave where fluid and solid travel at the same speed. The other one is a wave which is often damped far from the source due to the friction between fluid and solid as they no longer travel together. It is found that tortuosity plays an important role on the amplitude of the waves. The results were then compared to the predictions by Biot's theory for waves through porous media. Biot's theory is shown to be inaccurate in predicting the transient dynamic behaviour, but it is sufficient in capturing the overall trends. Finally, the model is used to predict waterhammer response in near wellbore reservoir.

1. Introduction

Coupling of fluid flow with geomechanics is necessary when analysing the propagation of pore pressure waves in porous media. Biot (1941) proposed the theory of poroelasticity which ignores the acceleration terms and wave effects. By wave we mean any discontinuity or jump in the field parameters such as pressure, temperature and stress (Hill, 1962). Later, Biot extended his formulations to elastic waves in saturated porous media for low frequency (Biot, 1956a) and high frequency waves (Biot, 1956b). A wave with low frequency is defined as the wave whose wavelength is less than the pore scale for which Poiseuille flow is valid (Sivrikoz, 2009). Different mathematical modelling is required at high frequencies since certain parameters such as permeability and tortuosity are frequency dependent. Biot's formulations have been used extensively in various applications including the study of the effects of earthquake shear waves on saturated sand response (Cheng, 1986; Desai and Galagoda, 1989) to the ultrasonic waves travelling in human bones (Lakes et al., 1986). The applications however focus on stress waves and ignore the effect of pore pressure waves in the fluid flow mainly because they are mostly damped and are of importance only around the source. Another reason is that Biot's theory predicts two waves in porous media while the second wave was not observed experimentally in porous media until 1980 (Plona, 1980) and later by van der Grinten et al. (1985) in soil. An example of pore pressure wave is waterhammer (WH) waves around an injector wellbore, the effects of which can be detrimental to the stability of wellbore and is suspected to induce sudden massive sand production because of sand liquefaction (Santarelli et al., 2000, 2011; Hayatdavoudi, 2005). The exposure of the reservoir to water hammer amplitudes that can move the stress conditions towards near-zero mean effective stress (or liquefaction conditions) poses a potential risk to the

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Nomenclature

a	speed of wave through solid
c	speed of wave through pipe
C_f	fluid compressibility
C_m	porous medium compressibility
C_s	grain compressibility
D	inner diameter of the pipe
E	Young modulus of pipe
G	shear modulus
I	identity matrix
k	permeability
K_s	bulk modulus of solid
K_f	bulk modulus of fluid
l	thickness of pipe
n	porosity
n_0	initial porosity
n_D	dimensionless porosity
p	pore pressure
p_0	initial pore pressure
p_a	applied pore pressure
p_D	dimensionless pore pressure
q_1	Landshhoff term in artificial viscosity damping
q_2	Von Neumann term in artificial viscosity damping
r	radial direction
S_p	storativity of the porous medium
t	time
$\boldsymbol{v}_r$	fluid velocity in r direction
$\boldsymbol{v}_z$	fluid velocity in z direction
$\boldsymbol{w}_r$	solid velocity in r direction
$\boldsymbol{w}_z$	solid velocity in z direction
z	vertical direction
α	Biot's coefficient
ϵ_v	volumetric strain
$\dot{\epsilon}$	strain rate
μ	fluid viscosity
ρ_f	fluid density
ρ_s	solid density
ρ_{12}	added mass density
τ	tortuosity
$\boldsymbol{\sigma}$	stress tensor
$\bar{\sigma}$	mean stress; $\bar{\sigma} = \sigma_{ii}/3$
θ	tangential direction

stability of the wellbore.

Verruijt (2010) presented an analytical solution using Biot's 1D dynamic formulation for a shock pore pressure wave and showed that two p-waves are generated as a result of a shock wave. He also verified the results with numerical simulations and obtained a reasonable match. The simulation results, however, showed numerical oscillations when shock waves were calculated.

de la Cruz and Spanos (1989) solved the thermodynamics of porous media for low-frequency seismic waves. They used continuity and momentum balance equations and added thermal coupling to the poroelasticity equations and treated porosity as a primary variable. In their formulation, they related velocities and deformations to heat generation of the second order and compression to heat generation of the first order. They showed that the heat flow leads to wave attenuation. Their formulation has been used in the mathematical demonstration of the feasibility of the application of pore pressure pulsing as an Enhanced Oil Recovery (EOR) method (Spanos et al., 1999).

Sivrikoz (2009) simplified the equations presented by de la Cruz and Spanos (1989) for pore pressure and solid displacements as the main variables under isothermal conditions, and solved them for the application of pressure pulsing as an EOR method. The method of solution adopts an elastic constitutive model to simplify the governing equations and is not applicable to elasto-plastic cases.

The work presented in this paper adopted the approach proposed by de la Cruz and Spanos (1989) for a saturated porous medium, ignoring the thermal effects by assuming isothermal conditions, assuming 2D axial symmetry, and employing artificial viscosity to damp the numerically-induced oscillations and achieve smooth response for shock waves. An elasto-plastic constitutive model was implemented to account for inelastic deformations. The state variables were chosen to be fluid velocity, solid velocity, pore pressure, porosity and stresses. The model was validated against experimental data published by van der Grinten et al. (1985) and van der Grinten et al. (1987). The results were also compared with those of Biot's formulation. The explicit finite difference method was used to solve the governing equations by employing a sequential coupling scheme combined with the velocity–stress method (Virieux, 1986).

2. Theory and background

de la Cruz and Spanos (1989) derived the governing equations for elastic solids by substituting stresses with displacements in Hook's law. The following equations are derived for isothermal conditions.

2.1. Mass balance equation for elastic medium

The derivation of mass balance is expressed in more detail to emphasize the assumption of elasticity in describing the solid response. Next section will discuss changes required on the governing equations for a more general constitutive model for solid. The mass balance equation for fluid is:

$$\frac{\partial(n\rho_f)}{\partial t} + Div(n\rho_f \boldsymbol{v}) = 0 \tag{1}$$

where n is the porosity, ρ_f is the fluid density and $\boldsymbol{v}$ is the fluid velocity vector. This equation is Eulerian while the solid momentum equation is usually expressed in the Lagrangian framework. Due to solid deformation, the change in mass will not be equal to the change in $n\rho_f$. Hence, the material derivative is introduced:

$$\frac{\partial()}{\partial t} = \frac{d()}{dt} - \boldsymbol{w}{\cdot}Grad() \tag{2}$$

where $\boldsymbol{w}$ is the solid velocity vector. The mass balance equation for the fluid can be rewritten as:

$$\frac{d(n\rho_f)}{dt} - \boldsymbol{w}{\cdot}Grad(n\rho_f) + Div(n\rho_f \boldsymbol{v}) = 0 \tag{3}$$

Also since divergence is a linear operator, it satisfies the product rule. Therefore, for any vector $\boldsymbol{F}$ and scalar a, one can write:

$$Div(a{\cdot}\boldsymbol{F}) = Grad(a){\cdot}\boldsymbol{F} + aDiv(\boldsymbol{F}) \tag{4}$$

Now if $a = n\rho_f$ and $\boldsymbol{F} = \boldsymbol{w}$, it yields:

$$\boldsymbol{w}{\cdot}Grad(n\rho_f) = Div(n\rho_f \boldsymbol{w}) - n\rho_f\, Div(\boldsymbol{w}) \tag{5}$$

Replacing $\boldsymbol{w}{\cdot}Grad(n\rho_f)$ in the mass balance equation: