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A modified multiscale finite element method for nonlinear flow in reservoirs

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ABSTRACT

In this paper we propose a modified multiscale finite element method for nonlinear flow simulations in heterogeneous porous media. The main idea of the method is to use the global fine scale solution to determine the boundary conditions of the multiscale basis functions. When solving the time-dependent problems, the equations of standard MsFEM need to be solved many times for different pressure profiles. Then, we propose an adaptive criterion to determine if the basis functions need to be updated. The accuracy and the robustness of the modified MsFEM are shown through several examples. In the first two examples, we consider single phase flow in consideration of pressure sensitivity and unsaturated flow, and then compare the results solved by the finite element method. The results show that the multiscale method is accurate and robust, while using significantly less CPU time than finite element method. Then, we use the modified MsFEM to compute two phase flow in low permeability reservoirs. The results show that the greater the staring pressure gradient is, the greater the pressure drop is, and the greater the staring pressure gradient is, the smaller the swept area is. All the results indicate that modified the MsFEM offers a promising path towards direct simulation of nonl-linear flows in porous media.

1. Introduction

A large number of scientific problems in a variety of areas involve nonlinearity. Nonlinear problems widely exist in different engineering practices, such as subsurface hydrology, civil engineering, petroleum engineering. Multiphase flow in low permeability reservoirs and unsaturated flow in heterogeneous porous media are examples of this type.

Low permeability reservoirs have become one of the main sources for oil and gas production. A lot of low permeability flow tests and production practice show that (Miller and Low, 1963; Lv et al., 2002; Wan et al., 2011): the minimum starting pressure gradient exists in the oil gas flow. Fluid in low permeability reservoirs must have an additional pressure gradient to overcome the rock surface adsorbed film to flow, the additional pressure gradient is known as the starting pressure gradient. The percolation curve deviates from the classical Darcy's law, with significant nonlinear characteristics. Aiming at this phenomenon, many experts have proposed many kinds of nonlinear percolation models (Huang, 1997; Xiao et al., 2007), and carried out the corresponding numerical simulation (Guo et al., 2004; Han et al., 2004; Xu et al., 2007; Yang et al., 2001). In general, finite difference techniques have been used in the petroleum industry for reservoir simulation applications. Such methods are based on the physical quantities of the central point of the adjacent element. It uses these quantities to construct numerical scheme, and then combined with the percolation curve to solve the problems. However, for the heterogeneous reservoir with low permeability, the percolation model of different elements are not the same. Numerical simulation must obtain the pressure gradient of each element, and select the corresponding flow model. Thus, the finite difference method is unable to meet the requirements. Jim Douglas and Todd DuPont (Douglas et al., 1969; Settari et al., 1977) made the innovative suggestion of using finite element techniques in nonlinear problems. Recently, a number of papers on the finite element methods have been published (Han et al., 2004; Zhao, 2006; Zhi et al., 2012; Tang et al., 2005). Compared to the finite difference methods, finite element methods are suitable for the problems with complex boundaries, high-order difference equation, and it can also be used in the heterogonous reservoir. Despite such favorable properties, use of the finite element methods in multiscale problems is severely restricted by the large computation and memory overheads. Thus, it is not surprising that researchers are looking for a method with a structure of finite element method to solve multiscale nonlinear problems.

Nonlinear flow in porous media are affected by heterogeneity of subsurface formations spanning over many scales (Tang et al., 2005; Huang et al., 2011; Weinan and Engquist, 2003). The

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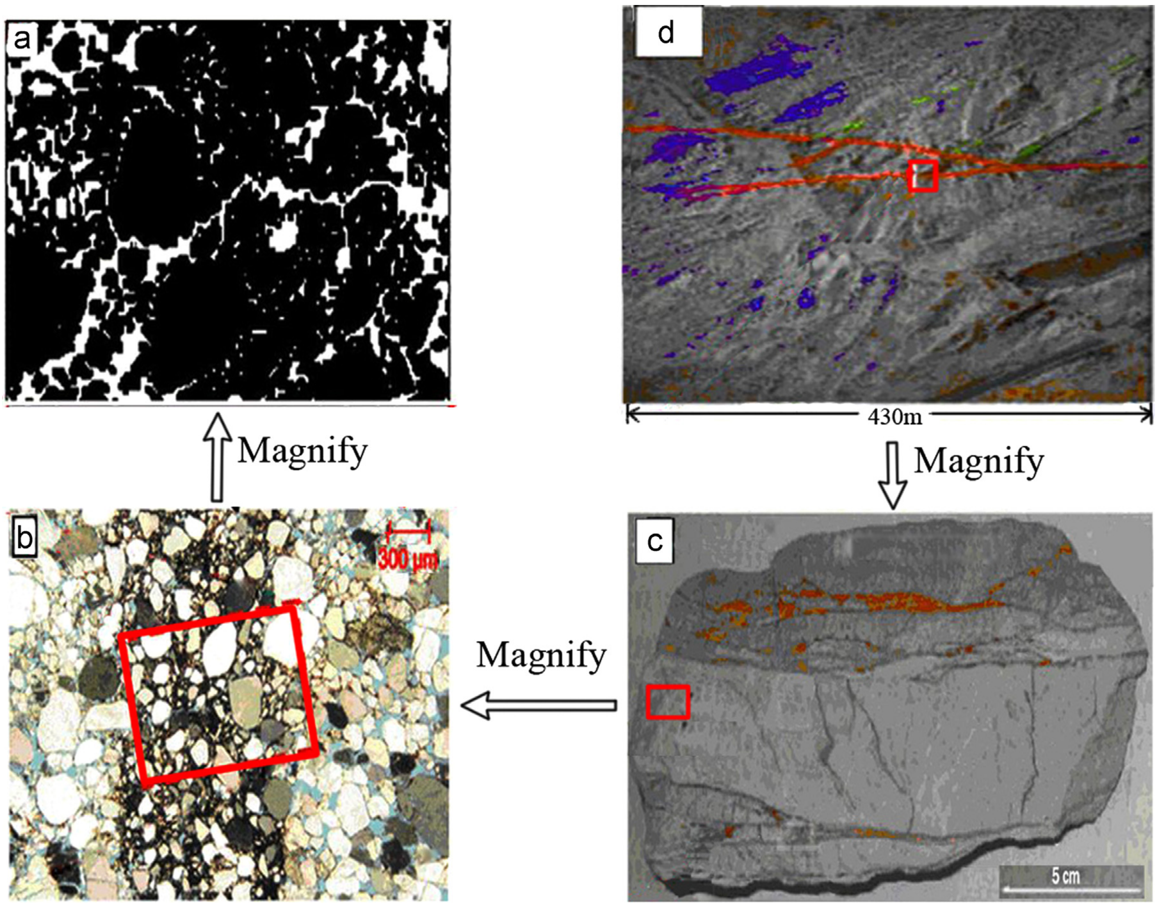

Fig. 1.1. Schemaic description of heterogeneities of different scales (Zhi et al., 2012).

heterogeneity is often represented by the multiscale fluctuations in the permeability (hydraulic conductivity) of the porous media (Hou and Wu, 1997). As shown in Fig. 1.1, the multiscale heterogeneities occur at a variety of scales, from microscopic to field scale. Therefore, the incorporation of the multiscale structure of the solution at all scales is important for numerical simulation. However, it is difficult to resolve numerically all of the scales even with supercomputers. For flow simulation, the geological model commonly include millions of grid blocks involved with each block having a dimension of tens of meters, whereas the permeability is at a scale of several centimeters (McCarthy, 1995). If using traditional finite element method or finite difference method (Huang et al., 2011; Rutqvist et al., 2002; Zhang, 2005) to solve such problems, it requires a tremendous amount of computer memory and CPU time and they can easily exceed the limit of today's computing resources. In practical calculation, typically, upscaling or multiscale methods are employed for such systems (Rizzi, 1976; Desbarats, 1998; Kueper and McWhorter, 1992; Efendiev and Hou, 2009). The main idea of upscaling methods is to form coarse-scale equations with a prescribed analytical form. These upscaling methods have proved quite successful. However, it is not possible to have a priori estimates of the errors that are present when complex flow processes are investigated using coarse models constructed via simplified settings. In multiscale methods, the fine-scale information is carried throughout the simulation and the coarse-scale equations are generally not expressed analytically, but rather formed and solved numerically. A number of multiscale numerical methods (Hou et al., 1999; Hughes et al., 1998; Efendiev and Hou, 2002; Aarnes et al., 2006; Juanes and Patzek, 2005; Chen and Hou, 2003) have been presented, such as dual mesh method, heterogeneous multiscale method (HMM) (Weinan and Engquist, 2003), multiscale finite element method (MsFEM) (Hou and Wu, 1997) and variational multiscale method (Hughes et al., 1998). Multiscale methods have proved to be capable of handling industry-standard complexity with respect to both grid representation and flow physics.

Traditional finite element method (FEM) was mentioned in nineteen fifties and sixties. Briefly, FEM is obtained in four steps: (1) discretize the domain into finitely many elements, and assume that all the elements are homogeneous; (2) construct finite element basis functions required to describe the physical characteristics of elements; (3) obtain the finite element formulation using various methods such as Galerkin method, and then assemble the local element equations into a global form and impose the boundary conditions; (4) Solve the global formulation (Reddy, 2006). Generally, the basis functions are linear or quadratic. When the element is heterogeneous and the variation of the physical field is nonlinear, the local element information is disregarded if using the polynomial basis functions, and this can lead to large errors (see Fig. 1.2(b)). If discretizing the domain more finely, the size of the discrete problem will be increased and it is more computationally expensive. Therefore, the application of FEM has been limited by the basis functions required to adequately describe the real physical processes.

MsFEM, which can be traced back to the work presented by Babuška and Osborn (1983) and Babuška et al. (1994), is first introduced by Hou and Wu (1997). This method is based on the construction of multiscale finite element basis functions that are adaptive to the local property of the differential operator, and was introduced as a tool to solve elliptic partial differential equations with multiscale solutions. It could capture the small effect on the large scale without resolving all the small scale details. Thus, it