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A new theoretical method to calculate shale fracture conductivity based on the population balance equation

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ABSTRACT

Shale fracture conductivity plays a critical role in determining the long term production of shale wells. High fracture conductivity is achieved by pumping more proppants and minimizing conductivity damage by fracturing fluid. Currently, fracture conductivity is either measured in laboratory following ISO 13503-5 or calculated using correlations derived from extensive laboratory tests by regression analysis. There is a need for a handy and practical tool to calculate shale fracture conductivity reflecting realistic fracturing designs.

This paper presents a new correlation to calculate shale fracture conductivity that considers proppant properties, fracture design variables and formation mechanical properties. This correlation is based on the population balance equation for size reduction. It utilizes the population balance concept to predict the crushed proppant size distribution under increasing closure stress. Numerical solution of the integro-differential equation is validated by comparing the computed results with the measured data. The crushed grain size distribution determines the permeability of the sand pack. Fracture width is calculated separately by considering proppant embedment and proppant grain rearrangement. Fracture conductivity is then calculated and compared with laboratory measured conductivity. Finally, the effect of water damage to shale fracture conductivity is considered by bringing in a new modification term.

Results show that the population balance equation can reasonably predict the crushed proppant size reduction by properly choosing the selection equation and breakage equation. The calculated shale fracture conductivity can match with the measured fracture conductivity using the Barnett Shale. An exponential expression can be used to account for the water damage effect in the Barnett Shale.

1. Introduction

More and more pilot projects in shale reservoirs have shown that shale well production is positively correlated with the amount of proppants pumped in hydraulic fracturing (LaFollette et al., 2014; Holcomb et al., 2015). This observation indicates that propped fractures with higher conductivity elevate shale well production. The industry uses ISO 13503-2 to standardize the manufacturing of hydraulic fracturing proppants and ISO 13503-5 to measure the long term fracture conductivity. The recommended practices regulate that fracture conductivity should be measured at the concentration of 2 lb/ft^2. Conductivities of different proppants in compliance with the ISO practices as well as mutated procedures to account for realistic conditions are utilized to select proppants by the operators. A huge amount of time and labors are required to measure the realistic fracture conductivity. To generate a useful correlation from experiments, it needs data from hundreds of laboratory tests. The regression method was typically utilized to generation a correlation with multiple variables (Wen et al., 2015). So, there is a need for a cost-effective and accurate theoretical model to predict the fracture conductivity.

One of the earliest fracture conductivity calculation equations was derived by Darin and Huitt (1959) from the Kozeny–Carman relation based on the partial monolayer assumption. In this equation, fracture width is an input calculated by geometric relations between proppant grain diameter and embedment depth (Huitt and Mcglothlin, 1958). The most widely used fracture conductivity calculation toolbox is a Stim-Lab product called Predict-K. This program contains the correlations found by the Stim-Lab consortium between fracture conductivity and proppant embedment, proppant crushing, gel filter cake and non-Darcy flow. These relations are established mainly by statistical method such as regression of extensive experimental datasets. Gao et al. (2012)

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Nomenclature

a_1–a_6	coefficients of the selection and breakage functions, dimensionless
B	breakage function, dimensionless
d_{10}	proppant diameter at the tenth percentile, L, mm
d_{50}	median proppant diameter, L, mm
d_{90}	proppant diameter at the ninetieth percentile, L, mm
e_0	initial void ratio, fraction
e	void ratio at certain stress, fraction
E	Young's modulus, $ML^{-1}T^{-2}$, MMpsi [pa]
m_P	weight percentage of the amount of proppants for size increment of dy
S	selection function, dimensionless
w_0	initial fracture width, L, m
w_1	fracture width at certain stress, L, m
w_f	fracture width, L, m
x	grain size, L, mm
y	dummy variable in the population balance equation
φ	porosity, fraction
φ_0	initial proppant pack porosity, fraction
φ_1	proppant pack porosity at certain stress, fraction
γ_P	proppant specific gravity, dimensionless
σ	closure stress, $ML^{-1}T^{-2}$, psi [pa]
ψ	phi percentile deviation, L, mm
i	index for the proppant grain size increment
n	index for the closure stress increment

derived a mathematical model to calculate the proppant embedment depth and fracture conductivity based on the Hertzian elastic contact theory. They matched the predicted results with experimental results by introducing a number of empirical coefficients. A new fracture conductivity correlation was proposed by Awoleke et al. (2012) using factorial design. This correlation links facture conductivity with proppant concentration, temperature, closure stress and gel loading. Unpropped fracture conductivity calculation has been well studied in acid fracturing where fracture conductivity is created by unevenly etched fracture faces. Conductivity correlations for unpropped rough fractures were discussed and derived by Nierode and Kruk (1973), Gong et al. (1999), Deng et al. (2012) and Zhang et al. (2014a, 2015a).

Water damage is a common mechanism of flow capacity loss in porous media (Bang et al., 2008). Remedial removal of water damage was studied both experimentally and theoretically (Yuan, 2012). Shale fracture conductivity damage by water is unique and significant in clay rich shale. Experiments show that over 80% of shale fracture conductivity measured in laboratory conditions can be lost after the samples are exposed to water flow (Zhang et al., 2014b). Thus, the water damage needs to be considered in shale fracture conductivity correlations.

Previous correlations to calculate fracture conductivity either require extensive laboratory tests to calibrate the model or depend on a single parameter that is difficult to measure under laboratory conditions. Besides, there is little published data to show that water damage is considered in the previous conductivity correlations.

This study intends to develop a cost effective and handy correlation that can be used to calculate shale fracture conductivity considering proppant properties, fracturing design variables, formation conditions and water damage effect.

2. Description of methodology

Fracture conductivity is calculated through the following steps:

(1) Review the recognized correlations for sand pack permeability calculation.
(2) Derive and solve the population balance equation for size reduction that describes proppant crushing in the fracture during elevated stress application.
(3) Validate the solution of the population balance equation by comparing the calculated results with the sieve analysis data.
(4) Measure the initial proppant pack width under zero closure stress and calculate the width change due to grain re-arrangement and embedment.
(5) Calculate the fracture conductivity and compare the results with the measured fracture conductivity.
(6) Modify the correlation that is based on the dry gas measurements to consider water damage.

3. Fracture permeability calculation

3.1. Sand pack permeability correlation

It has been recognized that sand pack permeability is a strong function of sand grain size, packing, and sorting. Kozeny (1927) developed a capillary model that accounts for shape factor in actual rocks to calculate the permeability of porous media. Carman (1937) extended Kozeny's model by introducing a factor for grain surface area to represent the irregular surface of pores. One of the most popular and accurate correlations to calculate sand pack permeability was proposed by Berg (1970):

$$k = 5.1 \times 10^{-6} \varphi^{5.1} d_{50}^{2} e^{-1.385\psi} \tag{1}$$

This model considers porosity of the porous media, grain particle size and sorting. However, one difficulty of applying this model to calculate propped fracture permeability is to determine the median grain size (d_{50}) and phi percentile deviation (ψ) of the crushed proppants, because proppant crushing is a dynamic process during the stress application. It is challenging to collect the low concentration proppants after experiment to do sieve analysis for particle size distribution. Even at higher concentration such as 0.20 lb/ft^2 where the total mass is 6.40 g, post experiment sieve analysis brings too much error due to the loss of proppants during the experiment, because some proppants are embedded into the shale matrix and cannot be retrieved. The used proppants are always blended with shale flakes or particles and cannot be separated due to the similar size ranges. More importantly, we are interested in permeability at different stress levels. We cannot measure size distribution at each stress level without disturbing the proppant pack. Therefore, a method to calculate particle size distribution at various stress levels is needed to determine fracture permeability.

3.2. Population balance equation for size reduction

A population balance equation describes the process of population changes of certain species as the net change of population is equal to the rate of concentration change. The net change of population is defined as the difference between species birth and death and they are usually functions of the entire population. So, population balance equations take the form of integro-differential