Journal of Petroleum Science and Engineering 128 (2015) 44–52

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

A simple and accurate numeric solution procedure for nonlinear buckling model of drill string with frictional effect

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ARTICLE INFO

Article history:
Received 9 April 2014
Accepted 17 February 2015
Available online 25 February 2015

Keywords:
Buckling
Horizontal well
Friction
Shooting method
Drill string

ABSTRACT

The nonlinear buckling analysis of drill string in a rigid well is presented in this paper. Considering effects of friction and boundary constraints, this problem could be taken as a model of a rod laterally constrained in a rigid cylinder (horizontal, oblique and vertical rigid cylinder could be included). After introducing a new variable, the resulting coupled nonlinear integral–differential equations are successfully solved by employing the extended system shooting method. Examples with various friction coefficients and combinations of boundary conditions are proposed. It is found that the axial frictional force plays a more significant part on buckling load for horizontal well than vertical one. Compared to experimental data or results obtained by using the discrete singular convolution algorithm (DSC) and the finite element method (FEM), the accuracy of the formulations and solution procedures, is verified. What's more, the nonlinear buckling behaviors of two instances of vertical scientific wells are analyzed. The present results are useful for practical design applications related to calculation of buckling loads and selection of bottom hole assembly (BHA) elements.

1. Introduction

This study of the structural buckling behavior of drill pipes is motivated by interest in many aspects of petroleum engineering (such as the functions of pipes, safety and surveying accuracy of down-hole instruments, etc.). Because of the high frequency of drill string failure, drill string lock-up, and casing wear, the stability of drill string has been a serious problem in oil/gas field operations for many years (Gulyayev et al., 2009; Tan and Gan, 2009). On the other hand, with the development of drilling technology, oil/gas and deep continental scientific drilling wells become very long currently, even more than ten kilometers. Furthermore, some oil/gas and deep continental scientific drilling wells may have very complex geometrical configurations, such that parts of wells may be inclined, vertical, horizontal, just plane curved, and even 3-D curved. Therefore, it is important and meaningful to investigate the buckling behavior of drill string for the science and technologies in petroleum engineering, deep continental scientific drilling and other related fields.

In despite of the complexity of the problem, many results are still reported. Paslay and Bogy (1964) first studied the problem of sinusoidal buckling of the tube. Based on the principle of minimum potential energy, the problem of helical buckling of a vertical tube was first analyzed by Lubinski et al. (1962). Since then, Cheatham and Pattillo (1984), He and Kyllingstad (1993), Miska and Cunha (1995) have studied helical buckling of tubes in vertical, horizontal or inclined wellbores, based on the energy method. Experimental study of helical buckling of a horizontal rod in a tube was performed by McCann and Suryanarayana (1994). While Wicks et al. (2008) reviewed available analytical and experimental results on the structural behavior of constrained horizontal cylinders subjected to axial compression, torsion, and gravity.

Frictional interactions with the constraining wellbore could cause buckling lock-up of drill strings, in which the drill strings are unable to progress, especially in large displacement horizontal wells. Understanding the post-buckling behavior of drill string, therefore, is important to avoid lock-up condition. Mitchell (1986), Wu and Juvkam-Wold (1993), Gao and Miska (2009) have took the friction effects into consideration in the theoretical analysis, while McCann and Suryanarayana (1994) study the frictional effects on buckling behavior by experimental method. Recently, Gan et al. (2009) use the differential quadrature element method to investigate effect of the gravitational and friction loads on buckling behavior of drill string, in order to overcome the shortage of the

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http://dx.doi.org/10.1016/j.petrol.2015.02.026
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local methods (such as the finite element method), and the global method (such as the differential quadrature method). By using the discrete singular convolution (DSC) algorithm, Wang and Yuan (2012) investigate the effects of friction and boundary constraints on the nonlinear buckling behavior of a relatively short rod constrained in a rigid horizontal cylinder and subjected to axial compression, gravitational and frictional loads. A modified version of DSC-based iterative scheme is presented to solve the coupled nonlinear integral–differential equations. They conclude that effects of friction on the buckling behavior are strongly depended on the boundary conditions for short drill pipes.

The purpose of this paper is to investigate the nonlinear buckling analysis of drill string in a rigid well by using the extended system shooting method. The accuracy of the formulations and solution procedures is verified, compared to experimental data or results obtained by using the DSC and FEM. Through analyzing the buckling deformation of drill strings in two specific wells, we find that the axial frictional force plays a less significant part on buckling loads for vertical well than horizontal one. The present results are useful for practical design applications related to calculation of buckling loads and selection of bottom hole assembly (BHA) elements.

2. Nonlinear buckling equations

For the title problem, a model of a drill string laterally constrained by a horizontal or vertical rigid well is schematically displayed in Fig. 1. Assume that the rod can rotate freely with respect to its axis, thus only axial friction force is encountered (Wicks et al., 2008). The rod is subjected to a compressive force P at its left end and a resulting compressive force $F_b = P + qL\ \cos\ \alpha - \mu \int_0^L W_n ds$ at its right end if the

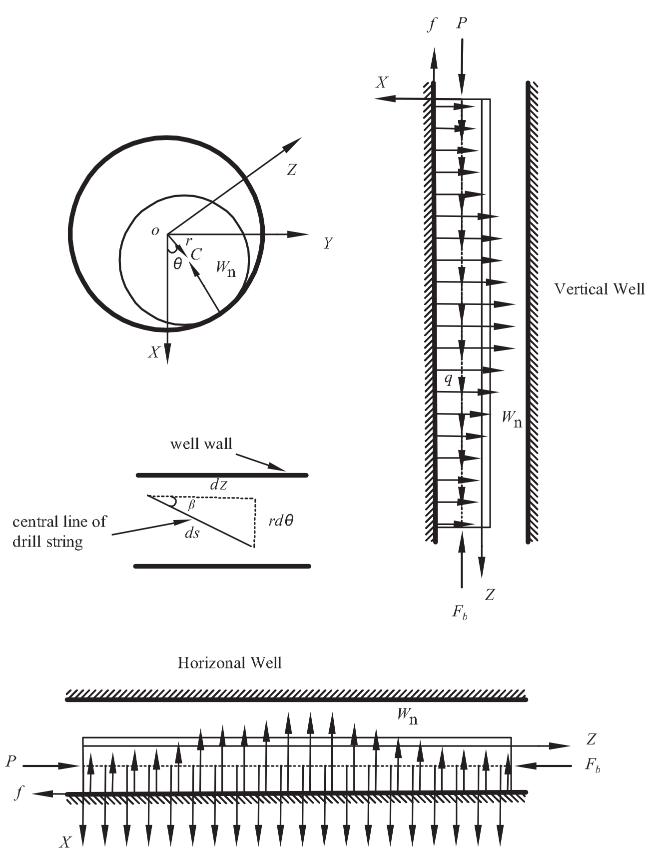

Fig. 1. Sketch of a drill string in a rigid well (including horizontal and vertical wells).

drag friction force is taken into considerations, here W_n is the contact force per unit length, and $\bar{f} = \mu W_n$ is the drag friction force per unit length acting opposite to the loading direction. Note that initially the rod laterally contacts with the cylinder to simplify the problem, although real boundary conditions would involve rod centralized in packers. A right-handed Cartesian coordinate system is set in Fig. 1. The drill string is assumed inextensible, which always contacts with the outer well during deformation. The buckling governing equation of a drill string with a wellbore can be expressed as in the following form (Mitchell, 1988; Gao et al., 2002; Wicks et al., 2008; Wang and Yuan, 2012)

$$EI\frac{d^4\theta}{ds^4} + F\frac{d^2\theta}{ds^2} + F'\frac{d\theta}{ds} - 6EI\left(\frac{d\theta}{ds}\right)^2\frac{d^2\theta}{ds^2} + \frac{q \times\ \sin\ \theta\ \sin\ \alpha}{r} = 0 \tag{1}$$

$$W_n - Fr\left(\frac{d\theta}{ds}\right)^2 - q \times\ \cos\ \theta\ \sin\ \alpha + EIr\left(\frac{d\theta}{ds}\right)^4 = 0 \tag{2}$$

where is the axial force (Wicks et al., 2008),

$$F = P + qs\ \cos\ \alpha - \mu\int_0^s W_n ds \tag{3}$$

The different boundary conditions are as follows:

$$\theta(0) = \theta(L) = \theta''(0) = \theta''(L) = 0 \quad \mathrm{S-S} \tag{4}$$

$$\theta(0) = \theta(L) = \theta''(0) = \theta'(L) = 0 \quad \mathrm{S-C} \tag{5}$$

$$\theta(0) = \theta(L) = \theta'(0) = \theta''(L) = 0 \quad \mathrm{C-S} \tag{6}$$

$$\theta(0) = \theta(L) = \theta'(0) = \theta'(L) = 0 \quad \mathrm{C-C} \tag{7}$$

where θ is the deviation angle of the drill string axis in the X, Y plane, and r is the radial clearance between the pipe and the wellbore. The material of the rod is isotropic with an elasticity modulus of E. The length, weight per unit length and bending rigidity of the rod are L, q and EI, respectively. The path of the deformed drill string centerline is $s \in [0, L]$, and μ is the frictional coefficient. The inclined angle of the drill string is α; the horizontal well is corresponding to $\alpha = 90^\circ$, while the vertical one is $\alpha = 0^\circ$. The helix angle β shown in Fig. 1 is small, therefore, could be approximated by

$$\beta \approx\ \sin\ \beta = r\frac{d\theta}{ds} \tag{8}$$

For details of the model derivation, readers are referred to Wang and Yuan (2012).

The path of the deformed drill pipe centerline could be calculated by the following formulae, once θ are obtained, (Yuan and Wang, 2012)

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} r\ \cos\ \theta \\ r\ \sin\ \theta \\ 0 \end{Bmatrix}_{s=0} + \int_0^s \begin{Bmatrix} -r\frac{d\theta}{ds}\ \sin\ \theta \\ r\frac{d\theta}{ds}\ \cos\ \theta \\ 1 - \frac{1}{2}r^2\left(\frac{d\theta}{ds}\right)^2 \end{Bmatrix} ds \tag{9}$$

where $u, v,$ and$w$ are components of the position vector of the deformed drill string centerline. For simply expressing, Eqs. (1), (2) (4)–(7) and (9) can be expressed in the following dimensionless form:

$$\theta^{(4)} + \left(\lambda + \bar{q}S\ \cos\ \alpha - \bar{\mu}\int_0^S N d\tau\right)\theta'' + (\bar{q}\ \cos\ \alpha - \bar{\mu}N)\theta' - 6(\theta')^2\theta'' + Q\ \sin\ \theta\ \sin\ \alpha = 0 \tag{10}$$

$$N - \left(\lambda + \bar{q}S\ \cos\ \alpha - \bar{\mu}\int_0^S N d\tau\right)(\theta')^2 + (\theta')^4 - Q\ \cos\ \theta\ \sin\ \alpha = 0 \tag{11}$$

$$\theta(0) = \theta(2\pi) = \theta''(0) = \theta''(2\pi) = 0 \quad \mathrm{S-S} \tag{12}$$