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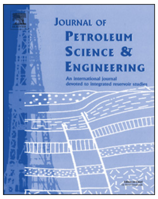

Laminar to turbulent transition of yield power law fluids in annuli

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ABSTRACT

An analysis of laminar to turbulent transition of yield power law (YPL) fluids in concentric and eccentric annuli is presented. Both theoretical and experimental approaches are followed to better understand the onset of transitional flow.

The objective of this study is to investigate the stability of the flow in concentric and eccentric annuli. Theoretical analysis of the inner and outer shear regions, to clarify the earlier transition observed with experimental studies, are within the scope of this study.

A stability criterion based on the ratio of turbulent energy production and rate of work done by viscous stresses is used to determine the end of laminar flow. Experiments are conducted for laminar, transition and turbulent regions of flow in a fully eccentric annulus. Eight distinct YPL fluids are tested and the results are compared with a proposed model and models available in the literature.

The proposed stability parameter shows an earlier transition near the outer wall for a wide range of non-Newtonian fluids, which is in agreement with measurements in the literature. The proposed modification is extended to eccentric annuli and showed good agreement with experiments. To the authors' knowledge, this is the first theoretical study to locally predict the onset of transition in eccentric annuli of YPL fluids.

Transition from laminar to turbulent significantly depends on eccentricity, diameter ratio and fluid properties, especially to the shear thinning ability of a fluid. The proposed modification allows a fair prediction of the transition from laminar to turbulent regions in eccentric annuli. With the proposed approach, the percentage of laminar and non-laminar regions for a cross section of an eccentric annulus can be predicted.

1. Introduction

Prediction of the transition from laminar to turbulent flow is a topic of interest for many industries, including the chemical process industry, food industry and oil industry. Depending on the machinery or application, laminar or turbulent flow may be desired. Accurate prediction of the transition from laminar to turbulent flow state is important both during the operation and the design phases.

In the drilling industry, an accurate estimation of frictional pressure losses during drilling is necessary to control the well, optimize bit hydraulics, prepare a proper drilling fluid program and select the appropriate pump. Precise estimation of annular pressure losses is vital, especially for offshore, multilateral and extended reach applications where the gap between pore and fracture pressure is usually narrow. To predict frictional pressure losses in a system, the flow state should be determined; i.e. laminar, transitional or turbulent flow. In each flow state, the prediction of frictional pressures losses will be handled with corresponding models.

Determination of the flow state is important for cementing and hole cleaning. During displacement of cement, laminar or turbulent flow maybe desired depending on well geometry, trajectory and fluid properties to ensure good mud removal and cement placement. Proper hole cleaning is crucial for successful drilling operations and better cuttings transport can be obtained with turbulent flow, especially in highly inclined and horizontal sections. Therefore, knowing the flow rate for the initiation of turbulence and accurate estimation of the flow state will provide optimized cementing and effective cuttings transport. Moreover, if the area and location of laminar and non-laminar regions in a cross section of eccentric annuli are predicted, the available hydraulic and cuttings transport models can be improved.

Most drilling fluids show non-Newtonian flow behavior and exhibit high shear-thinning performance. These fluids can be characterized as yield power law (YPL) fluids. There is limited information available for the transition of YPL fluids, especially in

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Nomenclature

a, b	geometric parameters (m)
D	diameter (m)
dP/dl	frictional pressure loss gradient (Pa/m)
dP	frictional pressure loss (Pa)
E	offset distance (m)
f	friction factor
h	height of the slot (m)
K	consistency index (Pa s^m)
m	flow behavior index
N	generalized flow behavior index
Q	flow rate (m^3/s)
r, R	radius (m)
Re	Reynolds number
V	mean fluid velocity (m/s)
w	width of the slot (m)
x, T_0	dimensionless yield stress
X	dimensionless coordinate for the flow in annuli

Greek letters

τ	shear stress (Pa)
γ	shear rate (1/s)
ε	dimensionless eccentricity
Θ	angle of the annular slice
λ	geometrical constant (m)
ρ	density (kg/m^3)
μ	viscosity (Pa s)
κ	diameter ratio
υ	velocity (m/s)

Subscripts

app	apparent
b	bulk
y	yield
o	outer
i	inner
w	wall
h, H	hydraulic
YPL	yield power law

annuli. The Herschel and Bulkley (1926) fluid model (YPL) is given as

$$\tau = \tau_y + K\dot{\gamma}^m \tag{1}$$

In this research, a theoretical and experimental approach is followed. A published stability parameter is modified to predict the end of the laminar region of YPL fluids, both in concentric and eccentric annuli throughout a cross-section. The experimental facility used in this research provides a wide range of flow rates (0–454 lpm), which allows observation of laminar, transition and turbulent regions for the tested fluids. Pressure measurements are taken for the laminar-to-turbulent transition with a relatively long, fully-developed test section (28 m). A proposed model, models from the literature and experiments are compared to assess precision of the models in predicting the onset of transition.

2. Literature review

Many models and approximations have been published to predict the end of the laminar region. There are various examples of studies on the transition from laminar to turbulent flow in pipes in the literature (Desouky and Al-Awad, 1998; Dodge and Metzner, 1959; Gucuyener and Mehmetoglu, 1996; Hanks, 1963; Hanks and Ricks, 1974; Maglione, 1995; Reynolds, 1883; Ryan and Johnson, 1959). Publications also exist for the transition in annuli; several authors (Dou et al., 2010; Hanks, 1963; Hanks and Bonner, 1971; Hanks and Ricks, 1974; Maglione, 1995) proposed stability criteria using a kinetic energy equation, while others (Founargiotakis et al., 2008; Gucuyener and Mehmetoglu, 1996; Guillot, 1990; Metzner and Reed, 1955; Reed and Pilehvari, 1993) used new Reynolds number definitions to predict transition from laminar to turbulent flow. Experimental studies have been conducted for the flow of Newtonian and non-Newtonian fluids in concentric annuli (Hanks, 1963; Hanks and Bonner, 1971; Hanks and Peterson, 1982; Japper-Jaafar et al., 2010; Rothfus et al., 1950) and Newtonian fluids in eccentric annuli (Bourne et al., 1968).

Osborne Reynolds (1883) developed a dimensionless parameter, which is named after him. The Reynolds number quantifies the competition between inertia and viscous dissipation during the flow of any fluid. For internal pipe flow, when the Reynolds number is less than 2100, laminar flow exists. When the Reynolds number exceeds the critical value of 2100, turbulent flow is present. The first experimental study on measuring the onset of transitional flow in concentric annuli was conducted by Rothfus et al. (1950). He tested air as a Newtonian fluid at room temperature with two different diameter ratios. It was reported that the smaller diameter ratio causes an earlier transition than the larger ratio. They observed a shift in the location of maximum velocity toward the inner pipe in the transition region. It was suggested that turbulence starts within the radius of maximum velocity while the rest is in viscous motion.

A dimensionless local stability parameter was presented by Ryan and Johnson (1959). The stability parameter is the ratio of turbulent energy production and the rate of work done by the viscous stresses. For Newtonian laminar pipe flow, when the parameter reaches a value of 808 laminar flow ends. It is claimed that the value of 808 determines the transition from laminar pipe flow to turbulent flow for non-Newtonian fluids. The Z definition is given as

$$Z = \left| \frac{r_w \rho \upsilon}{\tau_w} \frac{\partial \upsilon}{\partial r} \right| \tag{2}$$

The first stability criterion for concentric annuli was proposed by Hanks (1963). He suggested that if acceleration forces reach a certain multiple of the magnitude of viscous forces of the flow, distributions in flow starts and laminar flow ends. The onset of transition for Newtonian pipe flow is suggested to occur when the value of Hank's stability parameter reaches 404. The value is also claimed to be valid for non-Newtonian fluids, but Gucuyener and Mehmetoglu (1996) showed that this is not the case for Bingham Plastic fluids. If necessary simplifications are made, it can be seen that Hanks parameter includes Ryan and Johnson's parameter as a special case. The general form of Hanks criterion is given as:

$$K = \left| \frac{\rho \upsilon \mathrm{x} w}{\mathrm{div}\ \tau} \right| \tag{3}$$

where w is the vorticity vector ($w = \nabla \mathrm{x} \upsilon$) and τ is the deviatoric stress tensor. If the K profile is evaluated with its absolute value there will be one maximum and if the maximum reaches 404, the model predicts the end of laminar flow.

Stability of flow at the inner and outer regions of concentric annuli was investigated by Hanks and Bonner (1971). They suggested two