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High order approximation for scattering matrix in layered elastic medium and its application in pre-stack seismic inversion

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ABSTRACT

Scattering matrix of Plane P- and SV-wave in a layered elastic medium is the theoretical basis for pre-stack seismic inversion. Since the exact expression of scattering matrix is too complicated to apply, the current pre-stack seismic inversion method uses the first order approximation to estimate the stratigraphic elastic parameters, assuming the elastic parameter contrasts are small. It only works in weak reflection interface situation. When the elastic parameter contrasts become large, the first order assumption will suffer reduction in accuracy. At this time, the low order approximation should be adjusted with higher order corrections. Using series expansion, we infer the expression of any order approximation for scattering matrix. These high order terms are corrective terms adjusted to the first order approximation. Through different layer models, the comparison between the first, second, third order approximation reflection coefficient with the exact results of scattering matrix is done, and the accuracy of different order approximations is analyzed. In the following, using a 1D model, the pre-stack inversion based on different order approximations via nonlinear variable metric algorithm is performed. Inversion results illustrate the pre-stack seismic inversion based on higher order approximation not only adapts situations with weak reflection interfaces, but also adapts situations with strong elastic parameter variations on reflection interface. It breaks the limitation of small elastic parameter contrast assumption in conventional pre-stack seismic inversion.

1. Introduction

Pre-stack seismic inversion is important to indicate the stratigraphic elastic parameters, whose theoretical basis is the scattering matrix given by Aki and Richards (2002) which describes plane P- and SV-wave reflection and transmission in a layered elastic medium. Since the exact expression of scattering matrix is too complicated to apply in practice, many scholars give out different approximate expressions according to different aspects (Bortfeld, 1962; Richards and Frasier, 1976; Ostrander, 1984; Shuey, 1985; Smith and Gidlow, 1987; Zheng, 1991; Fatti et al., 1994; Verm and Hilterman, 1995; Gray et al., 1999; Aki and Richards, 2002; Russell et al., 2011; Gui et al., 2011; Zong et al., 2012a, 2012b). All the formulas are first order approximations based on the assumption that elastic parameter contrasts are small. These first order approximations only adapt to weak reflection interface, and will suffer a remarkable reduction in accuracy as the elastic parameter variation crossing reflect interface becoming large. It is difficult to obtain reasonable inversion results in this situation. To solve this problem, more accurate approximation for the scattering matrix with higher order is needed.

Considering the parameter variation, we present the method for solving scattering matrix to get more exact expression. And then, using Taylor series expansion with respect to the relative changes in velocity and density, we infer the higher order approximations for scattering matrix. In this paper, we present the first, second and third order approximation terms of reflection coefficient, which is generated by downward incident P-wave on an elastic interface. These approximations are commonly applied in pre-stack seismic inversion. Through different layer models, the comparison of the first, second and third order approximation reflection coefficient curves with exact results, and the accuracy analysis for different order approximations is carried out. Using a one-dimensional model which contains some strong reflection interfaces, pre-stack inversion based on different order approximations with nonlinear variable metric algorithm (Davidon, 1991; Fletcher, 2000) is performed. Comparing the first order to the high order inversion results, it can be seen that the pre-stack seismic inversion based on high order approximations adapts not

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only weak reflection interface situation, but also situations with strong elastic parameter variations on reflection interface. It breaks the limitations of conventional pre-stack seismic inversion due to the assumption of small elastic parameter contrasts and extends the application scope of pre-stack seismic inversion.

2. High order approximation for scattering matrix

2.1. Scattering matrix

Fig. 1 illustrates the reflection and transmission for an incident plane P- or SV-wave upon an elastic interface. Here, character P and SV respectively represent P-wave and SV-wave, with an accent mark indicating whether it is an upgoing wave (e.g.$\grave{p}$) or a downgoing wave (e.g.$\acute{P}$). $\alpha_i, \beta_i, \rho_i$ represent P-wave velocity, SV-wave velocity and density of the ith layer, where $i = 1, 2$. Aki and Richards wrote the scattering matrix for P–SV waves in a layered elastic medium as the following form Figs. 2–7

$$\begin{pmatrix} \grave{P}\acute{P} & \grave{S}\acute{P} & \acute{P}\acute{P} & \acute{S}\acute{P} \\ \grave{P}\acute{S} & \grave{S}\acute{S} & \acute{P}\acute{S} & \acute{S}\acute{S} \\ \grave{P}\grave{P} & \grave{S}\grave{P} & \acute{P}\grave{P} & \acute{S}\grave{P} \\ \grave{P}\grave{S} & \grave{S}\grave{S} & \acute{P}\grave{S} & \acute{S}\grave{S} \end{pmatrix} = M^{-1}N \tag{1}$$

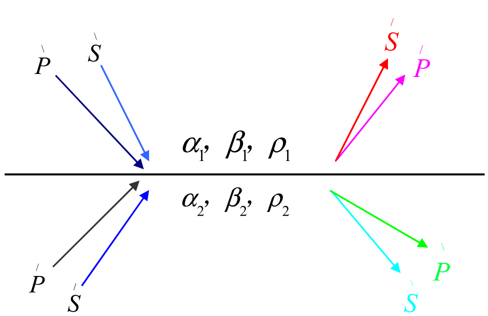

Fig. 1. Reflection and transmission for an incident plane P- or SV-wave upon an elastic interface.

where, M and N are coefficients matrix as the following

$$M = \begin{bmatrix} -\alpha_1 p & -\sqrt{1-p^2\beta_1^2} & \alpha_2 p & \sqrt{1-p^2\beta_2^2} \\ \sqrt{1-p^2\alpha_1^2} & -\beta_1 p & \sqrt{1-p^2\alpha_2^2} & -\beta_2 p \\ 2\rho_1\beta_1^2 p\sqrt{1-p^2\alpha_1^2} & \rho_1\beta_1(1-2\beta_1^2p^2) & 2\rho_2\beta_2^2 p\sqrt{1-p^2\alpha_2^2} & \rho_2\beta_2(1-2\beta_2^2p^2) \\ -\rho_1\alpha_1(1-2\beta_1^2p^2) & 2\rho_1\beta_1^2 p\sqrt{1-p^2\beta_1^2} & \rho_2\alpha_2(1-2\beta_2^2p^2) & -2\rho_2\beta_2^2 p\sqrt{1-p^2\beta_2^2} \end{bmatrix} \tag{2}$$

$$N = \begin{bmatrix} \alpha_1 p & \sqrt{1-p^2\beta_1^2} & -\alpha_2 p & -\sqrt{1-p^2\beta_2^2} \\ \sqrt{1-p^2\alpha_1^2} & -\beta_1 p & \sqrt{1-p^2\alpha_2^2} & -\beta_2 p \\ 2\rho_1\beta_1^2 p\sqrt{1-p^2\alpha_1^2} & \rho_1\beta_1(1-2\beta_1^2p^2) & 2\rho_2\beta_2^2 p\sqrt{1-p^2\alpha_2^2} & \rho_2\beta_2(1-2\beta_2^2p^2) \\ \rho_1\alpha_1(1-2\beta_1^2p^2) & -2\rho_1\beta_1^2 p\sqrt{1-p^2\beta_1^2} & -\rho_2\alpha_2(1-2\beta_2^2p^2) & 2\rho_2\beta_2^2 p\sqrt{1-p^2\beta_2^2} \end{bmatrix} \tag{3}$$

where p is the horizontal slowness and obeys Snell's law

$$p = \frac{\sin\,\theta_1}{\alpha_1} = \frac{\sin\,\theta_2}{\alpha_2} = \frac{\sin\,\varphi_1}{\beta_1} = \frac{\sin\,\varphi_2}{\beta_2} \tag{4}$$

In Eq. (4), $\theta_i$is the P-wave incident angle, or reflection and transmission angle in the ith layer, $\varphi_i$is the SV-wave incident angle, or reflection and transmission angle in the ith layer, where $i = 1, 2$.

The elements in scattering matrix are reflection coefficients and transmission coefficients. For example, $\grave{P}\acute{P}$ is the reflection coefficient of an upgoing P-wave from an incident downgoing P-wave.

Let$x = \sin\,\theta_1, a = \frac{\alpha_2}{\alpha_1}, b = \frac{\beta_2}{\beta_1}, c = \frac{\rho_2}{\rho_1}, \gamma = \frac{\beta_1}{\alpha_1}$and substitute them into Eqs. (2) and (3), Eqs. (5) and (6) can be obtained

$$M = \begin{bmatrix} -x & -\sqrt{1-\gamma^2x^2} & ax & \sqrt{1-b^2\gamma^2x^2} \\ \sqrt{1-x^2} & -\gamma x & \sqrt{1-a^2x^2} & -b\gamma x \\ 2\gamma^2x\sqrt{1-x^2} & \gamma(1-2\gamma^2x^2) & 2b^2c\gamma^2x\sqrt{1-a^2x^2} & bc\gamma(1-2b^2\gamma^2x^2) \\ 2\gamma^2x^2-1 & 2\gamma^2x\sqrt{1-\gamma^2x^2} & ac(1-2b^2\gamma^2x^2) & -2b^2c\gamma^2x\sqrt{1-b^2\gamma^2x^2} \end{bmatrix} \tag{5}$$

$$N = \begin{bmatrix} x & \sqrt{1-\gamma^2x^2} & -ax & \sqrt{1-b^2\gamma^2x^2} \\ \sqrt{1-x^2} & -\gamma x & \sqrt{1-a^2x^2} & -b\gamma x \\ 2\gamma^2x\sqrt{1-x^2} & \gamma(1-2\gamma^2x^2) & 2b^2c\gamma^2x\sqrt{1-a^2x^2} & bc\gamma(1-2b^2\gamma^2x^2) \\ 2\gamma^2x^2-1 & -2\gamma^2x\sqrt{1-\gamma^2x^2} & -ac(1-2b^2\gamma^2x^2) & 2b^2c\gamma^2x\sqrt{1-b^2\gamma^2x^2} \end{bmatrix} \tag{6}$$

This form of M and N was derived from Achenbach (1973), which was analyzed by some scholars in particular (Levin, 1986; Keys, 1989; Innanen, 2011). By Kramer's rule, Ewing et al., (1957)

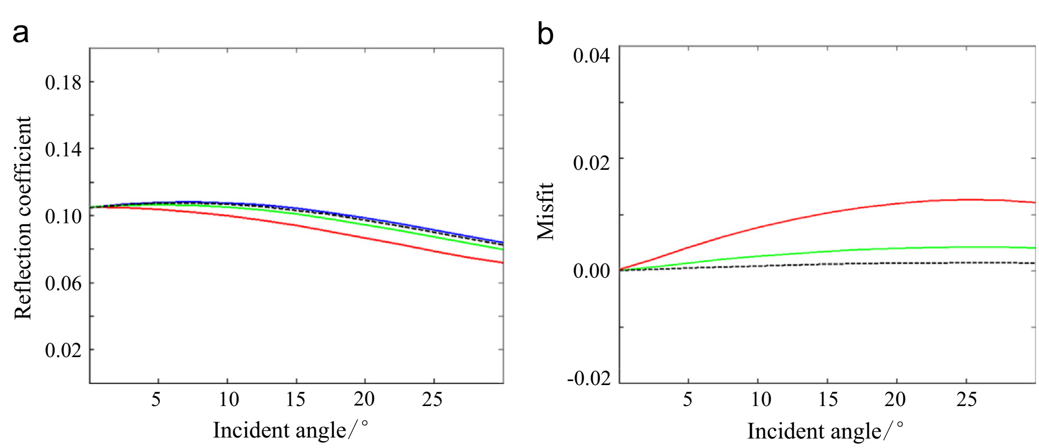

Fig. 2. Reflection coefficient curves (a) and misfit curves (b) for model 1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)