Journal of Petroleum Science and Engineering 132 (2015) 18–27

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Pareto-based robust optimization of water-flooding using multiple realizations

Elham Yasari, Mahmoud Reza Pishvaie *

Department of Chemical & Petroleum Engineering, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 22 May 2014
Accepted 29 April 2015
Available online 12 May 2015

Keywords:
Multi objective robust optimization
Water flooding
NSGA-II
Reservoir/formation
Uncertainty
Permeability
Small set of realizations

ABSTRACT

Robust optimization (RO) approach is inherently a multi-objective paradigm. The proposed multi-objective optimization formulation would attempt to find the optimum – yet robust – water injection policies. Two multi-objective, Pareto-based robust optimization scenarios have been investigated to encounter the permeability uncertainties. These multi-objective RO scenarios have been done based on a small representative set of realizations but they have introduced optimum points that could be reliable for the original set of realizations either. In both scenarios, the desired objective functions are expected value and variance of Net Present Value (NPV). The underlying RO scenarios have been done without any observation/measurement of pressures or well flows. Therefore, an ensemble of equally probable realizations has been used and ranked using Monte Carlo simulation technique. The Non-dominated Sorting Genetic Algorithm second version (NSGA-II) has been used as the optimization algorithm. The multi-objective robust optimization scheme has been applied for both scenarios via a twin setup of 100 realizations, one for investigation and the other one for validation purposes. The test studies demonstrated the superiority of the proposed methodology to give a robust optimal Pareto-based solution(s) (injection policies) under permeability uncertainties that could be reliable for the original set of realizations. Probability distribution functions (PDFs) of the original and small set of realizations have been depicted for comparison. Both optimization scenarios introduced optimum and robust injection policies that lead to higher expected value of NPV and lower variance, besides preserving the first and second moments of the original population of the original set of realizations.

1. Introduction

One of the main challenges in optimization of injected water rates is the spatial variability and uncertainty in formations (Mantoglou and Kourakos, 2007). Dealing with uncertainty is an important topic in all engineering applications such as petroleum production, reservoir optimization, water resources (Mantoglou and Kourakos, 2007; Ahmadi et al., 2010; Ren et al., 2013), ground water management field (Feyen and Gorelick, 2004; Baú and Mayer, 2007; Bayer et al., 2008), to name a few.

In the reservoir optimization problems, insufficient data leads uncertainty to intrinsic characteristics of a geological model (such as permeability and/or porosity map). Thus, the exact estimation of reservoir/formation properties will stay unknown and accordingly, optimization just based on a single and uncertain geological model would not be reliable. Applying various realizations of uncertain parameters with equal probability is an alternative way to rely on just one deterministic/stochastic value (van Essen et al., 2009; Alhuthali et al., 2010; Almeida et al., 2010; Chen and Hoo, 2012). This method is called Monte Carlo method and is a common way to deal with uncertain parameter values (Bayer et al., 2008; Carrera et al., 2005; Mantoglou and Kourakos, 2007; Baú, 2012). A single objective optimization has been considered in the mentioned works. There has been some multi-objective optimization within the petroleum engineering literature. But, most of the applications of these approaches appear to be in the area of history matching, where various measures of misfit have been used as the objective functions (Hajizadeh et al., 2011; Shelkov et al., 2013; Mohamed et al., 2011; Ferraro and Verga, 2009; Sayyafzadeh and Haghighi, 2012).

Multi-objective optimization has also been applied for field development problems, where objectives such as recovery factor, duration of plateau production, NPV, the voidage replacement ratio etc,… have been used as the objective functions (Gross, 2012; Sibaweihi and Awotunde, 2012; Isebor and Durlofsky, 2014). In our previous work (Yasari et al., 2013) a well control optimization

* Corresponding author. Tel.: +98 2166166429.
E-mail addresses: e_yasari@che.sharif.ir (E. Yasari), pishvaie@sharif.edu (M.R. Pishvaie).

http://dx.doi.org/10.1016/j.petrol.2015.04.038
0920-4105/

problem involving three objectives has been considered to optimize the different components of Net Present Value (NPV) under economical and geological uncertainty.

None of the reported works have considered robust optimization objectives (mean and variance), independently. Also, despite of the noticeable computational intensity of the optimization approach based on multiple realizations, in the reported works under uncertainty all the generated realizations have been used that leads to a huge computational time. On the other side, the reliability of the RO results based on the selected set of realizations has not been investigated. In this paper, we have tried to improve computational time and solving performance of the RO problems when using multiple sampled realizations by theoretical support of optimal multi-objective scheme. In order to encounter the time consuming process of RO, optimization has been done based on the small set (sampled set) of realizations as the representative of the original set of realizations. The small set of realizations has been selected using the statistical distribution of the economical objective function (NPV). This small set has been chosen based on the nine percentile of NPV data (Yang et al., 2011). Then, Multi-objective Robust Optimization (MORO) schemes have been performed based on this small set of realizations.

However, the results of the optimization based on this small set should be reliable for the original population either. Therefore, two multi-objective RO scenarios have been proposed to determine optimum points with this goal and PDFs of the small and original set of realizations have been depicted for comparison. The first multi-objective optimization formulation would attempt to find the optimum and robust water injection policies that lead to *higher* expected values of NPV and *lower* variances.

In this study, original data distribution is almost normal. So, in the second optimization problem preserving the first and second moments of the original population in the small set of realizations would be suffice. In this way, calculated results based on the small set of realizations would be reliable.

The general goal of RO is obtaining an optimum design value (water injection rate in each injection well as the decision variable) which is least sensitive to the uncertainty. Non-dominated sorting genetic algorithm second version (NSGA-II) with some modification to handle the constraints has been used as the optimizer engine.

Section 2 presents methodology of the work including Multi-Objective Robust Optimization, Pareto optimality, geological uncertainty, problem formulation and RO flowchart. In Section 3, the proposed approach is applied and verified via a twin setup of 100 realizations, one for investigation and the other one for validation. Further, in Section 4 the results are shown and discussed, followed by the conclusion in Section 5.

2. Methodology

2.1. Multi-objective robust optimization (MORO)

To cope with the uncertainty in the reservoir, a number of possible realizations that are equally probable would be generated and RO would be performed based on these possible realizations. In this work, permeability values are supposed to be the main sources of uncertainty in the process of water-flooding. The manipulated or decision variables would be computed and then applied for all realizations. There are several types of objective functions that could be utilized and in the present contribution, the optimization was performed by considering the 'expected sense' (Terwiesch et al., 1994).

Robust objective function(s) – The main aim of an RO algorithm is to exteremize the desired objective function value under uncertainties. This leads to defining two different objectives associated with the function to be optimized; mean value of the desired objective function ($\bar{f}$) and its variance δf (or standard deviation, $\sqrt{\delta f}$) defined by the following equations:

$$\bar{f} = E[\underline{f}] \approx 1/n_t \sum_{j=1}^{n_t} f_j \tag{1}$$

$$\delta f = [E[\underline{f}^2] - E^2[\underline{f}]] \approx \frac{1}{n_t - 1} \sum_{j=1}^{n_t} (f_j - \bar{f})^2 \tag{2}$$

where n_t denotes the number of realizations and $\underline{f}$ is a vector of desired objective functions. It should be noted that the square of $\underline{f}$ (i.e.,$\underline{f}^2$) is the element-by-element (Schur) product of $\underline{f}$ with itself. In this study there are 100 realizations (n_t) and NPV of each realization (f_j) has been considered as an element of the vector function $\underline{f}$.

RO is inherently a multi-objective scheme. As mentioned earlier, there are two main (conflicting) objective functions being formulated to deal with uncertainty and therefore, the problem becomes a multi-objective optimization problem.

Generally speaking, a multi-criteria optimization problem can be demonstrated as:

$$\begin{aligned} &\text{Minimize/Maximize} : \{f_1(\underline{u}), f_2(\underline{u}), \ldots, f_i(\underline{u}), \ldots, f_N(\underline{u})\} \qquad i = 1, \ldots, N \\ &\text{Subject to} \quad g_j(\underline{u}) = 0 \qquad j = 1, \ldots, M \\ &h_k(\underline{u}) \le 0 \qquad k = 1, \ldots, K \end{aligned} \tag{3}$$

where $\underline{f}(\underline{u})$ defined as $\{f_1(\underline{u}), f_2(\underline{u}), \ldots, f_i(\underline{u}), \ldots, f_N(\underline{u})\}$ is a vector of objective functions, $f_i(\underline{u})$ is a typical objective (or value) functions, and $\underline{u}$ is a vector of design or decision variables ($u_1, u_2, \ldots, u_i, \ldots, u_n$). N is the number of conflicting objective functions, M is the number of equality constraints, and K is the number of inequality constraints and n is the number of decision variables. NPV, productivity or water cut (and so on) are some objectives that could be defined as the objective functions in a multi objective optimization problem. In the current RO problem, the mean ($\bar{f}$) and variances (δf) (2 conflicting objective functions, N=2) of NPVs ($\underline{f}$) are considered as the multi objective optimization functions. Bottomhole pressures or water injection rates of each well are two examples of decision variables.

2.2. Pareto optimality

The solutions in the Pareto front (or set) are the ones that simultaneously satisfy all the objectives and there is no other solution that dominates them. Because finding the Pareto optimal front is very challenging, difficulties arise when conventional methods are used for these types of problems (Malekmohammadi et al., 2011). Expression (4) defines Pareto optimality for a two objective problem. For maximization problem, $\underline{\mathbf{x}}_1$ is said partially greater than $\underline{\mathbf{x}}_2$ if and only if:

$$\forall i: \quad f_i(\underline{x}_1) \ge f_i(\underline{x}_2) \quad and \quad \exists_i : f_i(\underline{x}_1) > f_i(\underline{x}_2), \quad i = 1, \ldots, N \tag{4}$$

$\underline{x}_1$ dominates the solution $\underline{x}_2$, if expression (4) holds.

When there is a trade-off between objectives this leads to a set of optimal solutions called the "Pareto set".

In Fig. 1 the continuous curves of Pareto front have been shown for four different scenarios with two objective functions. Each objective is able to be minimized or maximized. Solutions of Pareto optimal set are at the edge of the feasible search region (Burke and Kendall, 2005). It could be seen from Fig. 1 that minimizing or maximizing each objective defines a different problem with a different Pareto optimal front.

2.3. Geological uncertainty

In this study, the permeability map of reservoir has been considered as the uncertainty space. By discretizing this space, 100