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A novel approach for automatic grid generation for multi-phase flow simulators: A robust computational framework for mass transfer at bubble point pressure

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ABSTRACT

Coarsening from fine geological model to dynamic coarse scale is an essential task in reservoir simulation. In this paper a new approach is introduced to create a coarse dynamic model based on effective parameters of fluid flow in porous media. In this technique, the effects of fine scale permeability map, key flow paths (streamlines) and well location are considered to build a coarse dynamic model from a fine geological one. An exact element size map is generated by comparing all the mentioned effective parameters and making the element size indicator by selecting the maximum value. This element size map (background grid) is applied to build an unstructured mesh for discretization of the reservoir. Afterward, this intelligent mesh generator was employed in three-phase flow simulation. Moreover, a novel computational framework is developed for the evaluation of bubble point pressure to prevent divergence of the solution when the reservoir conditions change. To evaluate the performance of the developed model, the fluid flow rate obtained by this model is compared to that obtained by the uniform grid model. It is found that the proposed method provides more accurate results for fluid flow rate compared to the uniform grid model. Moreover, it is faster and computationally less expensive than the fine model. This model can be applied in reservoir simulators and provides more accurate and reliable results in less CPU time compared to the traditional mesh generation techniques. In addition, the proposed model solves the problem of divergence of the solution at the bubble point pressure.

1. Introduction

Knowledge of transport processes and accurate calculation of multi-phase flow play an important role in reservoir simulation. Study of multi-phase flow in porous media involves non-linear and coupled partial differential equations (PDE's) which requires a robust algorithm for the solution. Many studies have been conducted to analytically solve PDE's of multi-phase flow in porous media; however, the main problem of all analytical methods is that they consider the flow in porous media as completely immiscible (Juanes and Patzek, 2004; Shearer and Trangenstein, 1989; Souza, 1992). Strictly hyperbolic, non-strictly hyperbolic and mixed hyperbolic and elliptic (Holden, 1990) are the main analytical approaches. To solve PDE's, some parameters are required to characterize the fluid, media and their interactions. To this end, two algorithms can be utilized namely direct and indirect methods. In the direct methods, these parameters are obtained using analytical and semi-analytical approaches. On the other hand, the indirect algorithms are more consistent with experimental data of actual cases (Yeh, 1986). Peaceman (1983, 1987) extensively studied these algorithms for solving the equations including off-center and multiple wells within a well-block, non-square grids, anisotropic permeability, horizontal wells, and other general geometries.

Besides the analytical approaches, there are several numerical methods for solving multi-phase flow problems. Finite difference (FD) approach, which is widely used, has low computation cost and is strongly adaptive for structured grids. Staggered-grid finite difference (SGFD) method improves the accuracy of the results, and reduces the dispersion effects of the solution. Hassanzadeh (1991) solved the second order equation of Biot poroacoustic equations with FD scheme. This method which is actually used for regular grids, may lead to numerical dispersion. Thus, for the cases such as multi-scale structures or interfaces with high contrast between physical parameters, the grid

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Nomenclature

P	pressure
∇p	pressure gradient
u	velocity
ρ_α	density of the component α
S_α	Saturation of the phase α
μ_α	viscosity
u_α	volumetric velocity
B_α	formation volume factor
$k_{r\alpha}$	relative permeability of phase α
P_{COW}	capillary pressure of oil and water
e_{ij}	boundary
WI	well index
R_s	solution gas to oil ratio
$Q_{w,k}$	volumetric flow rate
P_b	bubble point pressure
D	depth function
P_{bh}	bottom hole pressure
φ_0	initial porosity
λ	mobility for each phase
ϕ	system porosity
ν_1	interstitial velocity of oil
ν_{cr}	critical value of oil interstitial velocity
S_d	desired element size map (Eq. (8))
S_a	applied element size map (Eq. (8))
x_i	inlet coordinate system positions in x direction
x_e	exit coordinate system positions in x direction
x_o	local coordinate system positions in x direction

size should be small enough to assure FD stability conditions. However, this method may lead to additional computation cost.

Finite volume schemes consist of three different methods, namely Implicit Pressure-Explicit Saturation (IMPES), Fully Implicit and Sequential methods among which IMPES scheme is the fastest; however, it may result in an unstable solution algorithm. In order to efficiently deal with irregularly geometrical and geological characteristics, there exists different discretization techniques. First, Aziz (1993) introduced these techniques. Heinrichs (1987) developed perpendicular bisector method (PEBI). Control volume finite element approach was then developed by Forsyth (1989) and applied for thermal simulation of a reservoir. Fung et al. (1991) developed this technique for other commercial reservoir simulators. Verma and Aziz (1997) applied this technique to three-dimensional systems. This method was also applied for black oil reservoir simulation with irregular shaped grids, and Li et al. (2004a, 2004b, 2003) discussed the stability of this method using control volume finite element method (CVFE). Control volume function approximation (CVFA) method is a suitable technique which is used for interpolation and flux computation. The priority of CVFA method over CVFE is the application of non-polynomial functions (e.g., spline, bilinear and weighted distance functions) which results in establishing flux continuity for arbitrary shaped control volumes and reducing the grid orientation effects (Li et al., 2003).

All of the above-mentioned FD and FV methods diverge at bubble point pressure or converge with so many iterations. Although the application of the unstructured meshes based on finite element and finite volume discretization techniques results in flexibility in computation process, it causes some additional efforts for management of mesh quality and computational overheads. Since there is deficiency in the previously published techniques for mesh generation, this paper aims to propose a novel approach for generating background grid.

In this study, a robust method for generating unstructured grid along with using spline interpolation function has been described and applied to discretize the flow equations. Then, this method was applied for generating an unstructured mesh which establishes flux continuity in flexible mesh structures and updates the reservoir properties in a simple and computationally less expensive approach. In this mesh generation technique, static and dynamic characteristics of the reservoir are precisely studied. Furthermore, application of the proposed method in multi-phase flow simulation has been investigated. As known, bubble point pressure is a critical point in multi-phase simulation of black oil. To prevent the divergence of the solution at this point, the proposed model accurately estimates the initial guess for the unknowns of the set of flow equations by solving a series of material balance equations.

2. Modeling approach

2.1. Permeability filtering

Wavelet–like kernels are used for detection of harsh jumps in input functions. In this function the area under the curve is zero which means that a constant function under this filtering would result in zero. For more details in this area refer to the work of Hesami and Dabir (2011). For a 2D case, this procedure is applied to both rows and columns of a two dimensional discrete function. This generates two energy values that the maximum is stated as the element size indicator. The mathematical formulation is developed in the work of Hesami and Dabir (2011):

$$\begin{aligned} &E_{M*N} = [E_{i,j}] \\ &E_{ij} = \max(E^R_{i,j}, E^C_{i,j}) \\ &E^R_{i,j} = \sqrt{(K^{RW}_j)^2 + +(K^{RH}_j)^2} \\ &E^C_{i,j} = \sqrt{(K^{CW}_i)^2 + +(K^{CH}_i)^2} \end{aligned} \tag{1}$$

$$\begin{aligned} &K^{RW}_{1*N} = \left[K^{RW}_{1*N}\right] = R_{1*N}*W \\ &K^{RH}_{1*N} = \left[K^{RH}_{1*N}\right] = R_{1*N}*H \\ &R_{1*N} = [K_{i,n} : n = 1, \ldots, N] \end{aligned} \tag{2}$$

$$\begin{aligned} &K^{CW}_{1*M} = \left[K^{CW}_{m}\right] = C_{1*N}*W \\ &K^{CH}_{1*M} = \left[K^{CH}_{m}\right] = C_{1*M}*H \\ &C_{1*M} = [K_{m,j} : m = 1, \ldots, N] \\ &K_{M*N} = [K_{m,n}] \end{aligned} \tag{3}$$

K_{m*n} is the input discrete function. R and C are the rows and transposed columns of the input function. W and H are the discrete 1D wavelet kernel and its Hilbert transform and $E^R_{i,j}, E^C_{i,j}, E_{i,j}$ are the row, column and final energy measures. A set of x–y separable functions and reconstruction formula are shown as follows (Hesami and Dabir, 2011):

$$\begin{aligned} &G_{2a} = 0.9213(2x^2 - 1)e^{-(x^2+y^2)} \\ &G_{2b} = 1.843(xy)e^{-(x^2+y^2)} \\ &G_{2c} = 0.9213(2y^2 - 1)e^{-(x^2+y^2)} \\ &G^\theta_2 = \cos^2(\theta)G_{2a} - 2\cos(\theta)\sin(\theta)G_{2b} + \sin^2(\theta)G_{2c} \end{aligned} \tag{4}$$

In which G_{2a}, G_{2b}, G_{2c} are base functions and G^θ_2 is the oriented second derivative of Gaussian function in θ direction. A set of x–y separable base is introduced to approximate the complementary H_2