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Unified maximum particle size prediction for turbulent dilute dispersions

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ABSTRACT

A new maximum stable particle size model for turbulent dilute dispersion flow is proposed, which includes both droplets and bubbles. The proposed model incorporates, for the first time, the combined effects of the dispersed-phase density and viscosity. The model has been tested against a database consisting of 169 sets corresponding to turbulent dilute dispersions showing a good agreement.

1. Introduction

In the oil and gas industry, the estimation of droplet and bubble size distribution is crucial for proper design and performance prediction of transport and processing systems. Some of these applications are listed as follows:

(1) Pal (1987) experimentally investigated the effect of the droplet size on the rheological behavior of oil–water mixtures. The results showed that droplet size strongly influenced rheology. Finer emulsions present larger viscosity as compared to coarse emulsions.

(2) Water droplets can be dispersed in the hydrocarbon phase, thus hydrates form at the hydrocarbon–water interface. Total water conversion into hydrates depends on the droplet size. Sloan et al. (2011) notes that for a droplet diameter larger than 40 μm hours or days may be required for full conversion.

(3) Corrosion in oil and gas pipelines is related to multiphase flow parameters such as water wetting/entrainment. Nesic et al. (2004) proposed a corrosion model based on the hydrodynamic characteristics of oil–water flow. An estimation of the maximum droplet size (related to breakup and coalescence) and critical droplet size (related to settling and separation) is utilized to determine the stability of water in oil dispersion.

(4) Separator performance is a strong function of dispersed-phase volume fraction and fluid particle (droplet or bubble) size distribution at the separator inlet. Thus, estimation of droplet and bubble size becomes a key parameter for separation design and performance. This is more critical for compact separation units, which are characterized by a short operational window, in terms of inlet bubbles and droplets size distribution.

State of the art in fluid particle size prediction defines two possible modeling approaches: *droplet size evolution* and *steady-state droplet size*. Droplet size evolution is an initial value problem and population balance equations are used to predict how the droplet size distribution evolves through a particular device. This approach is able to predict droplet change in time and space and has been successfully implemented in different applications (see Ramkrishna, 2000). This approach required the implementation of closure relationships to solve the fundamental equations. This relationship has been developed for particular cases and their generalization is still on a preliminary basis.

The second approach is the steady-state droplet size, which is the equilibrium point where the coalescence rate is equal to the breakup rate in a particular flow field. The key parameter of this approach is the prediction of the maximum stable droplet or bubble diameter. In turbulent flow, despite the complexity of particle deformation and breakup, some simple scaling relationships have been utilized to quantify the possible maximum particle diameter for a given system. The most commonly used model is based on the analysis of Hinze (1955), who postulated that the local shear stress deforms a fluid particle, which will breakup if this stress exceeds the force resisting deformation (surface tension). Hinze (1955) did not discriminate among types of breakup which might occur: primarily inertial, shear, shape oscillation, etc. Instead, he suggested that the local turbulent kinetic energy was an important parameter for the breakup process. Furthermore, Hinze (1955) proposed that the primary length-scale is of the same order as the particle diameter (d). The particle can be broken by eddies only in the same order of magnitude of the droplet or bubble size. Any particle smaller than the integral-scale of the turbulence will not be deformed by integral-scale fluctuations, which

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will instead simply transport the particle. The maximum particle size proposed by Hinze (1955) is given by

$$d_{MAX} = We_{CRIT(Hinze)} \left(\frac{\sigma}{\rho_C}\right)^{3/5} \varepsilon_0^{-2/5}, \tag{1}$$

where d_{MAX} is the maximum particle diameter of a dilute system, σ is the surface tension, ρ_C is the continuous phase density, ε_0 is the turbulent energy dissipation per unit of mass and $We_{CRIT(Hinze)}=0.725$ is the critical Weber number proposed by Hinze (1955). A disadvantage of the Hinze (1955) model is that it does not consider dispersed phase density variations, especially for bubbles where the operational pressure varies significantly.

Levich (1962) considered the balance between the internal pressure of a bubble and the capillary pressure of a deformed bubble. The dispersed-phase density was included through the internal pressure force term, and the capillary pressure was determined from the shape of the deformed bubble rather than a spherical bubble. Thus, Levich's (1962) critical diameter yields

$$d_{MAX} = We_{CRIT(Levich)} \frac{\sigma^{3/5}}{\rho_D{}^{1/5} \rho_C{}^{2/5}} \varepsilon_0{}^{-2/5}, \tag{2}$$

where ρ_D is the dispersed phase density. Hesketh et al. (1991) proposed a critical Weber number for the Levich (1962) formulation as $We_{CRIT(Levich)} \approx 1.1$. Eq. (2) predicts the maximum size of both bubbles ($\rho_D < \rho_C$) and droplets ($\rho_D < \rho_C$). The three previous approaches do not account explicitly for the effect of the viscosity of both the dispersed and continuous phases. Therefore, Hinze (1955) and Levich (1962) are strictly valid for dispersed-phase viscosities smaller than or equal to the continuous-phase viscosity, i.e., $\mu_D \le \mu_C$. For these conditions, the particle fragmentation is dominated by the pressure forces associated with the velocity fluctuations, where the viscous forces can be neglected (Kolmogorov, 1949).

Extension of Hinze (1955) to include the viscosity of the dispersed-phase was presented by Davis (1985) who added a viscous force term, yielding

$$d_{MAX} = We_{CRIT(Davis)} \left(\frac{\sigma}{\rho_C} + \frac{\sqrt{2}}{4} \frac{\mu_D (\varepsilon_0 d_{MAX})^{1/3}}{\rho_C}\right)^{3/5} \varepsilon_0{}^{-2/5}, \tag{3}$$

where μ_D is the dispersed phase viscosity. In summary, none of the available models for predicting the maximum stable fluid particle diameter include the effects of both the dispersed-phase density and viscosity. In this study, a novel and unified model (applicable to droplets and bubbles) for predicting the maximum fluid particle size in turbulent dispersion flow is proposed, including the density and viscosity of the dispersed phase.

In this paper, a comprehensive mechanistic model is proposed to predict the fluid particle distribution for turbulent flow considering the steady-state particle size approach. The proposed model requires the determination of the maximum stable diameter, a mean diameter and particle size distribution equations.

2. Unified maximum particle model

The starting point is the definition of the critical Weber number, which is given by the ratio between the disruptive and cohesive stresses. The disruptive stress is an inertial stress, while the cohesive stress is the sum of the interfacial and viscous stresses. In this study, Marangoni effects and interfacial elasticity are not considered; thus, the interfacial stress is only due to interfacial tension. The final configuration of deformed particle is closed to ellipsoidal shape. The mathematical description of this configuration is complex for the purpose of this study. Thus, cylindrical shape is considered, with height h, cross-sectional area A and volume $V=Ah$, the interfacial stress can be approximated by

$$\tau_{INTERFACIAL} = \frac{\sigma}{d_{MAX}} \sim \frac{\pi h^2}{V} \sigma. \tag{4}$$

It is proposed that the rate of deformation of a cylindrical droplet is of the magnitude of the turbulent fluctuation (u'_C). The viscous stress of the deformed particle can be calculated as

$$\tau_{VISCOUS} = \frac{\pi h^2}{V} \frac{\mu_D u'_C}{4}. \tag{5}$$

Considering a uniform temperature distribution inside and outside the particle, the energy balance on the particle is a function of pressure (p), interfacial tension and viscous forces work, as given by

$$\Delta pA\, dh + 4\sigma\, dA - \mu_D \frac{u'_C}{h} A\, dh = 0. \tag{6}$$

Due to the constant droplet volume and evaluating pressure difference across a fluid particle by applying Bernoulli equation around the surface, the energy equation can be simplified as follows:

$$h = \frac{2(4\sigma + \mu_D u'_C)}{\rho_C u'_C 2}. \tag{7}$$

Substituting Eq. (7) in Eqs. (4) and (5) and utilizing an equivalent spherical diameter to express the particle volume, the cohesive stress becomes

$$\tau_{COHESIVE} = \frac{96}{(d_{MAX})^3 (\rho_C u'_C 2)^2} (4\sigma + \mu_D u'_C)^3. \tag{8}$$

Levich (1962) postulated that the fluctuating velocities of both the surrounding fluid and fluid within the particle are equal. Thus, the disruptive stress becomes

$$\tau_{DISRUPTIVE} = \rho_D u'_C 2. \tag{9}$$

Finally, the critical Weber number is given by

$$We_{CRIT} = \frac{\rho_D{}^{1/3} \rho_C{}^{2/3} u'_C 2 d_{MAX}}{(\sigma + (\mu_D (u'_C/4)))}. \tag{10}$$

Furthermore, the eddies that cause the breakup of a particle have the same size as the particle ($d_{MAX} \approx \lambda$). Substituting in the Kolmogorov length scale results in

$$\overline{u'^2} = C(\varepsilon_0 d_{MAX})^{2/3}, \tag{11}$$

where according to Batchelor (1953) $C=2$. Substituting Eq. (11) into Eq. (10) and solving for the maximum fluid particle diameter yields the final developed expression for the maximum droplet size in dilute systems, namely

$$d_{MAX} = We_{CRIT} \left\{ \frac{[\sigma + 2^{-3/2} \mu_D (\varepsilon_0 d_{MAX})^{1/3}]^{3/5}}{\rho_D^{1/5} \rho_C^{2/5}} \varepsilon_0{}^{-2/5} \right\} = We_{CRIT} \Psi. \tag{12}$$

Eq. (12) incorporates, for the first time, the combined effects of the dispersed-phase viscosity and density ratios.

3. Comparison with experimental data

The collected literature data for fluid particle breakup in turbulent flow for dilute systems are summarized in Table 1. As shown, it includes 169 data points for different flow configurations, including