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Burst pressure analysis of pipes with geometric eccentricity and small thickness-to-diameter ratio

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ABSTRACT

Two analytical models were proposed to predict the burst pressure of pipes characterized by a small thickness-to-diameter ratio and geometric eccentricity. Based on a complex stress function in a bipolar coordinate system, an analytical solution was derived for the stresses of an internally pressurized eccentric pipe, and two burst pressure prediction formulae for restrained and end-capped pipe models were obtained according the von Mises stress, which took the ultimate tensile strength (UTS) of the inner wall as the pipe bursting criterion. Next these formulae were validated by comparing the results obtained with the results of existing elastic–plastic formulae and a nonlinear finite element method. Finally, the effect of eccentricity on the burst pressure was evaluated and simplified as an empirical formula. Our research shows that the burst pressure of restrained and end-capped pipes with geometric eccentricity are nearly equivalent, and the burst pressure of an eccentric pipe is relative to that of a non-eccentric pipe. The theoretical and empirical formulae presented in this paper have broad application in the oil and gas industry to predict the burst pressure of pipes.

1. Introduction

Cylindrical vessels and conduits, such as pipelines, subjected to high internal pressure are widely used in various industrial applications. However, pipelines can burst under the conditions of excessive internal pressure or plastic deformation when the pressure is just over the maximum pressure capacity of a pipeline. As such, industrial accidents related to bursting pipelines often occur, especially for pipes of small thickness-to-diameter ratio. A small thickness-to-diameter ratio λ usually refers to a pipe characterized by $\lambda < 0.1$, including some seemingly thick-walled pipes that are widely used in the oil industry and thin-walled pipes that are widely used in the rocket and missile industry. With the development of deep well and ultra-deep well oil drilling activities, drill strings and casings are bearing increasingly higher pressures, and the emergence of multibranch wells and horizontal wells makes eccentric wear increasingly prominent. Simultaneously, with the development of space technology, lightweight rocket and missile systems must accommodate varying degrees of eccentricity caused by manufacturing errors, wear and corrosion, etc. Hence, research and accurate prediction of the burst pressure of pipes characterized by a small λ is crucial in the fields of oil, natural gas, launch vehicles and missile systems design and reliability assessment.

Two main types of burst failure for cylindrical vessels under internal pressure have been identified, i.e. elastic–plastic fracture and brittle fracture. This paper focuses on the first type of failure. The development of predictive models related to the burst pressure of cylindrical vessels and pipes has been in progress for more than half a century (Xian-Kui and Brian N., 2012). Over that time, a large number of experimental, analytical and numerical studies have provided numerous theoretical and empirical formulae to predict burst pressure. However, a general equation for accurately predicting burst pressure that is widely accepted has not yet been provided (Law and Bowie, 2006, 2007).

Existing burst pressure prediction formulae are classified into three categories. The first category is based on the classic yield criteria – the von Mises criterion (the fourth strength theory) – which can be applied not only in the elastic range but also in the plastic range. These predication methods include the Nadai (1) (Bailey, 1930), Nadai (2) (Nadai, 1931), Soderberg (Soderberg, 1941), Faupel (Faupel, 1956), maximum stress (Christopher et al., 2002), API (American Petroleum Institute, 1992), etc. The second category is based on another classic yield criterion – the Tresca criterion (the third strength theory) – where pertinent methods

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Nomenclature

c	focal point of the bipolar coordinates in the rectangular coordinate system
c_0, c_1, c_2	coefficients in Eq. (31)
$D(D_o), D_i, D_m$	external diameter, internal diameter and mean diameter, respectively
e	2.718, the natural logarithm base
f_0, f_1, f_2	coefficients in Eq. (31)
g_0, g_1, g_2	coefficients in Eq. (27)
h_0, h_1	coefficients in Eq. (27)
k	R_o/R_i
n	strain hardening exponent: $n=\exp(1+\varepsilon_{ult})$, where ultimate strain ε_{ult} corresponds to ultimate tensile strength σ_{ult}
p_b, p_i, p_o	burst pressure, internal pressure and external pressure, respectively
$p_b^{e1}, p_b^{e2}, p_b^{p3}, p_b^{p4}$	elastic burst pressure for a restrained pipe, elastic burst pressure for an end-capped pipe, elastic–plastic burst pressure based on the Tresca strength criterion and elastic–plastic burst pressure based on the von Mises strength criterion, respectively
R_o, R_i, R_s	outer radius, inner radius and critical radius of elastic and plastic zones, respectively
$t, t_{\max}, t_{\min}$	wall thickness, maximum wall thickness and minimum wall thickness, respectively
x, y, z	components of the Cartesian coordinate system
YT	yield-to-tensile ratio
α, β	components of the bipolar coordinate system
α_i, α_o	internal boundary and external boundary, respectively
ε	eccentricity ratio δ/t
ε_u	uniform strain
ε_{true}	true strain
γ	modified coefficient $\gamma\approx 1$ for thin-walled pipes $(0<\lambda\le 0.02)$ and $\gamma\approx(0.8\sim 1)$ for pipes with somewhat thicker walls $(0.02<\lambda\le 0.1)$
δ	distance between the centres of inner and outside circles representing eccentricity
ζ	variable quantity in the bipolar coordinate system
η	angle between the Cartesian coordinate and bipolar coordinate
λ	ratio of thickness to diameter t/D
μ	Poisson ratio of the pipe material
$\sigma_{ys}, \sigma_{flow}, \sigma_{ult}, \sigma_{true}$	yield stress, flow stress, ultimate tensile strength (UTS) and true stress, respectively
$\sigma_\alpha, \sigma_{\alpha i}, \sigma_{\alpha o}, \sigma_\beta, \tau_{\alpha\beta}$	radial stress, inner radial stress, outer radial stress, hoop stress and shearing stress, respectively
$\sigma_1, \sigma_2, \sigma_3, \sigma_M$	the first, second and third principal stresses and von Mises stress, respectively

include the Turner (Turner, 1910), Bailey–Nadai (Nadai, 1950), ASME (Boiler and Pressure Vessels Code, 1962), maximum shear stress (Christopher et al., 2002), etc. The third category represents methods based on average models. Steward and Klever (Klever, 1992; Stewart et al., 1994; Klever and Stewart, 1998) indicated that a wide zone exists between the von Mises criterion and the Tresca criterion and proposed an average model based on both criteria, which reduced burst pressure prediction errors. Subsequently, Zhu and Leis (2004, 2004, 2006) proposed another average model based on the new multi-axis plastic yield theory known as the average shear stress yield criterion, or simply as the ZL (or Zhu–Leis) criterion, which improved the representation of the flow response from the elastic domain into and through the elastic–plastic domain. However, applying this criterion to determine the pipeline burst pressure remains problematic. For example, geometric eccentricity caused by manufacturing errors, wear, corrosion, etc. will reduce burst pressure. However, few related studies are available in literature (Huang et al., 2007).

In the present paper, an analytical method is used to study the effect of geometrical eccentricity on the pipeline burst pressure for pipes with a small λ. First, various existing pressure pipeline burst prediction formulae are listed and classified. An analytical, easy-to-use solution applicable to eccentric pipe geometry for restrained and end-capped pipe models under internal pressure is subsequently given, and the related burst prediction formulae are presented. These formulae are validated by comparing the results obtained with those of existing elastic plastic formulae and the nonlinear finite element method (FEM). Finally, the effect of eccentricity on the burst pressure for pipes with a small λ is analysed. In short, the formulae proposed in this paper demonstrate high accuracy, reliability and are widely applicable.

1.1. Summary of pressure pipeline burst prediction formulae

Twelve theoretical and empirical prediction formulae for the burst pressure p_b applicable for pipes characterized by a small λ are summarized in Table 1. The prediction formulae are classified into three categories in this study. In Table 1, Faupel, API, Nadai (1), Nadai (2), Soderberg and Maximum stress represent the first category based on the von Mises strength criterion. Turner, ASME, Maximum shear stress and Baily–Nadai represent the second category based on the Tresca strength criterion. Klever and Zhu–Leis represent the third category based on the von Mises strength and Tresca strength criteria.

2. Theoretical analysis of the eccentric pipeline burst pressure

2.1. Geometric description of an eccentric pipe

Depending on the manufacturing accuracy, the cross-section of a new pipeline will appear more or less as an eccentric ring, as shown in Fig. 1. In addition, wear, corrosion and other causes will exacerbate the eccentricity in the application. Fig. 1(a) shows a geometric model of a pipe with an inner wall defect. The dashed circle represents the nominal inner wall of the eccentric pipe, which describes a pipe with a theoretical inner diameter AA_0, that shares the same centre O_0 with the outer wall, although the actual inner wall has a centre O_1 and a diameter AA_1. Fig. 1(b) shows another geometric model of a pipe with an outer wall defect, where the dashed circle with diameter BB_0 and centre O_0 represents the nominal outer wall of the pipe in the absence of eccentricity, and the actual eccentric outer wall has a centre O_2 and diameter BB_1. However, regardless of whether the condition is that of an inner wall or outer wall defect, the cross-section of the eccentric pipe can be simplified into an area enclosed by two perfect circles with different centres and diameters, as shown in Fig. 2. To describe the pipe eccentricity and the variation of wall thickness, we define a parameter ε as

$$\varepsilon=\frac{t_{\max}-t_{\min}}{t_{\max}+t_{\min}} \tag{1}$$