Journal of Petroleum Science and Engineering 124 (2014) 173–179

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

A new methodology for analyzing non-Newtonian fluid flow tests

Fahd Siddiqui *, Mohamed Y. Soliman, Waylon House

Texas Tech University, 2500 Broadway, Lubbock, TX 79409, United States

ARTICLE INFO

Article history:
Received 31 October 2013
Accepted 12 October 2014
Available online 19 October 2014

Keywords:
Non-Newtonian
Pressure transient analysis
Power law fluids
Water flooding
Polymer injection
Fluid flow

ABSTRACT

Conventional treatment of oil flow in porous media assumes that oil is in the typical range of flow velocity and pressure, and it behaves as a Newtonian fluid. This assumption, however, may not be accurate, especially for heavy oil. It is also not accurate in modeling injection behavior for fluids such as fracturing fluids, or polymer fluids for secondary recovery of oil.

The literature contains articles that formulated and presented solutions of the posed fluid flow problem. When trying to apply the proposed solution methodology, the limitations of those techniques became apparent. Many authors have already pointed out the inapplicability of the presented techniques and suggested some corrections. Moreover the presented techniques are not valid for non-Newtonian fluids that have a power law index (n) greater than 0.6.

In this paper we examine the solutions presented in the literature showing their limitations. We will examine the mathematical reasons for the problems and demonstrate rigorously that these (published) analysis techniques are impractical not only for real world problems but also for simulated data. Our in depth analysis explains away the problems analysts have had with the existing methodology. We also present a new reliable methodology for determining reservoir properties. The new methodology was tested on six real field data and two simulated datasets. The examples showed that the previously presented methodologies under-predict the permeability by at least 40%.

1. Introduction

Non-Newtonian fluids have a viscosity that is a function of shear rate. The viscosity of such fluids exhibits a non-linear relationship between shear stress and shear rate. Fig. 1 shows the behavior of different types of non-Newtonian fluids compared to Newtonian fluids. In general all liquids exhibit a non-Newtonian character at certain shear rates, with Newtonian fluids being a sub class of non-Newtonian fluids. Common examples of such fluids in petroleum are polymer solutions. Eq. (1) is the model of power law applied to non-Newtonian fluids.

$$\tau = H\dot{\gamma}^n \tag{1}$$

Modeling the flow of non-Newtonian fluids through porous media has always been a challenge. Odeh and Yang (1979) and Ikoku and Ramey (1979) separately developed similar partial differential equations and proposed similar analysis techniques for practical well test analysis. Eq. (2) is the PDE proposed by Ikoku and Ramey

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{n}{r_D}\frac{\partial p_D}{\partial r_D} = r_D^{1-n}\frac{\partial p_D}{\partial t_D} \tag{2}$$

Ikoku and Ramey solved this PDE and gave two analysis techniques that are valid for $n<0.6$. However, Vongvuthipornchai and Raghavan (1987) showed that the solutions to the diffusivity equation require a significant correction factor when $n<0.6$. They compared these solutions against numerical solutions and found that the results are not in agreement and specifically this difference increases as n decreases.

Vongvuthipornchai and Raghavan also demonstrated that the Cartesian plot of p_D vs. t_D (as suggested by Ikoku and Ramey, and Odeh and Yang) results in straight lines that have a slope which is different from the slope of the straight lines resulting from numerical solution. This in turn means that an incorrect value of n can be deduced if the analysis techniques of these authors are used. However this difference vanishes as n tends to unity.

McDonald (1979) proposed a finite difference method of solving the PDE proposed by Odeh and Yang and found that a large number of nodes are required to get accurate results (when compared to the analytical solution proposed by Odeh and Yang).

Later studies (Pascal, 1991; Ciriello and Di Federico, 2012) have derived the analytical solutions to the above PDE for a moving

* Corresponding author.
E-mail address: fahdsiddiqui@gmail.com (F. Siddiqui).

http://dx.doi.org/10.1016/j.petrol.2014.10.007
0920-4105/

Nomenclature

c_t	total compressibility, Pa^{-1}
h	formation thickness, m
H	consistency (power law parameter), Pa s^n
k	permeability, m^2 [1 $m^2 = 1.013 \times 10^{15}$ md (milli Darcy)]
K_ν	modified Bessel function of the second kind of order ν
n	flow behavior index (power law parameter)
p	pressure, Pa
p_D	dimensionless pressure drop, non-Newtonian fluid
p_D'	dimensionless pressure derivative, non-Newtonian fluid
p_i	initial pressure, Pa
q	flow rate, m^3/s
r_D	dimensionless radial distance
r_w	wellbore radius, m
s	van Everdingen–Hurst skin factor
t_D	dimensionless time
t_{DNN}	dimensionless time, non-Newtonian
z	Laplace parameter
$\Gamma(x)$	gamma function
λ_{eff}	effective mobility, m^{n+1}/Pa s
μ_{eff}	effective viscosity, Pa s^n m^{1-n}
φ	porosity

injection front. These studies have used this solution to perform uncertainty analysis on power law index and porous medium properties towards the position of the front. However an analysis technique for determining the said properties, using pressure transient data, has not been fully developed from those solutions.

2. Analysis of the problem

The problem of non-Newtonian fluid flow was considered in order to formulate a general flow equation for porous media. Much has been published about the solutions of the problem of injecting polymers into the formation and the governing equations derived by various authors (Ikoku and Ramey, 1979; Mirzadjanzade et al., 1971; Odeh and Yang, 1979). This paper looks into the solutions presented by Ikoku and Ramey, and Odeh and Yang. Because both of the studies formulated a similar flow equation and give similar analysis techniques, they will be handled together.

Vongvuthipornchai and Raghavan (1987) have already pointed out problems with the analyses presented by these authors. We will examine the mathematical reasons for the problems and demonstrate rigorously that these (published) analysis techniques are impractical not only for real world problems but also for simulated data.

Ikoku and Ramey (1979) formulated the flow equation as given in Eq. (2). They derived a solution for constant rate and infinite reservoir (in Laplace space) at the wellbore as:

$$\bar{p}(z) = \frac{K_{(1-n)/(3-n)}(2/(3-n)\sqrt{z})}{z^{3/2}K_{2/(3-n)}(2/(3-n)/\sqrt{z})} \quad (3)$$

They then produced an approximate analytical solution by making an argument about long time approximation as:

$$p_D \approx \frac{(3-n)^{2(1-n)/(3-n)} t_D^{(1-n)/(3-n)}}{(1-n)\Gamma(2/(3-n))} - \left(\frac{1}{1-n}\right) \quad (4)$$

This solution has the assumption that the Bessel function can be approximated asymptotically as:

$$K_\nu(z) \sim \frac{1}{2}\Gamma(\nu)\left(\frac{1}{2}z\right)^{-\nu} \quad (5)$$

By inspection the problem arises with Eq. (4), because as $n \to 1$ Eq. (4) begins to diverge. The approximation presented by the authors in Eq. (5) is also invalid. Fig. 2 plots the Bessel functions and their approximations for various values of n and z. It is evident that even with z as small as 10^{-6} (real time as large as 3 days assuming typical values for formation and fluid properties) the approximation in Eq. (5) produces an error of 10% for $n = 0.6$.

Ikoku and Ramey provide two solution techniques. One technique involves plotting Δp vs. $t^{(1-n)/(3-n)}$ which should result in a straight line. The problem with this analysis technique is that time is raised to the power $(1-n)/(3-n)$ which varies between 1/3 and 0 for values of n between 0 and 1 respectively. Therefore if $n = 0.5$, the analyst is forced to raise time to the power of 0.2 which results in data compression on a Cartesian paper and multiple non-unique

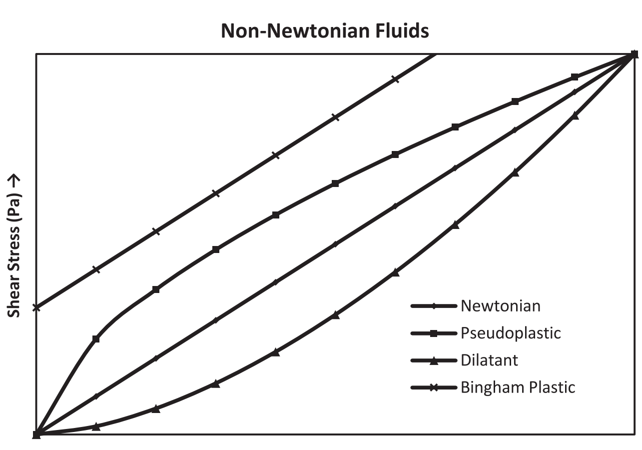

Fig. 1. Non-Newtonian fluids.

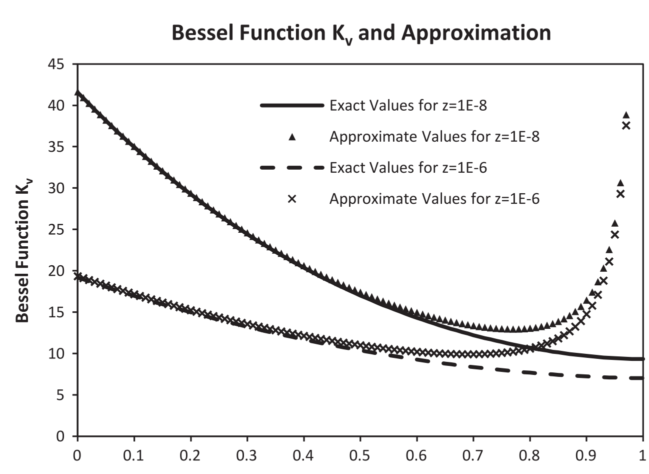

Fig. 2. Ikoku and Ramey (1979) approximation to Bessel function.