Journal of Petroleum Science and Engineering 124 (2014) 402–405

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Comment on "Gene expression programming analysis of implicit Colebrook–White equation in turbulent flow friction factor calculation"

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ARTICLE INFO

Article history:
Received 10 January 2013
Accepted 1 December 2013
Available online 12 December 2013

Keywords:
explicit approximations
Colebrook friction factor
Darcy–Weisbach equation
turbulent flow
fixed point method

ABSTRACT

Recently, it is investigated (Samadianfard, 2012) the potential of genetic programming based technique in estimating flow friction factor in comparison with the most currently available explicit alternatives to the Colebrook's equation.

Using iterative solution of the Colebrook's equation which is accurate to six significant digits, this discussion showed that the proposed approximation by the author for friction factor is not very accurate (the errors increase up to 7.374%), thus it is proposed two new accurate approximations (with maximum error less than 0.022% and 0.008%) for estimating flow friction factor.

1. Introduction

The discusser would like to thank the author for proposing an approximation for estimating flow friction factor using genetic programming based technique. The discusser, however, would like to add a few points.

Colebrook (1938–1939) developed the following implicit equation that combines experimental results of turbulent flow in smooth and rough pipes:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{2.51}{\mathrm{Re}\sqrt{f}} + \frac{\varepsilon/D}{3.71}\right) \tag{1}$$

in which f is the friction factor, Re is the Reynolds number, and ε/D is the relative pipe roughness. The Colebrook's equation is valid for the Re ranging from 4×10^3 to 10^8 and for the ε/D ranging from 0 to 0.05.

For developing an accurate explicit expression of the friction factor and for the analysis of error distribution, Eq. (1) should accurately be solved for various known values of Reynolds number and relative roughness. For this purpose, a rectangular space of the wide range of ε/D and Re values is considered. This area is covered with 1701 points (21×81 grid of ε/D and Re values) which uniformly are distributed in the logarithmic space. The practical range for the Reynolds number is considered between 10^4 and 10^8 while range for the relative roughness is between 10^{-6} and 0.1. All friction factors and errors shown in this note are calculated using the iterative fixed-point method where the coefficients in the Colebrook's equation are considered as 2.51 and 3.71. The accurate friction factors with six significant digits are presented in Table 1.

Percentage relative error of friction factor is defined as $\mathrm{RE}(\%) = 100\times|1-f_{estimated}/f_{accurate}|$, in which $f_{accurate}$ is the accurate friction factor that is obtained from iterative solution, Table 1, and $f_{estimated}$ is friction factor calculated using the selected explicit approximation of the Colebrook's equation. Distribution of percentage relative error of proposed approximation by the author (Eq. (29); Samadianfard, 2012) is computed and presented in Fig. 1. This approximation is not very accurate and errors increase up to 7.374%. This may be attributed to inaccurate solution of implicit Colebrook's equation by the author which is used in gene expression programming (GEP).

The author also missed very relevant references. As indicated in the study by Brkic (2011), there are five most accurate approximations to the Colebrook's equation with the maximum relative error of no more than 0.15%. The author did not compare his approximation with all of these approximations. For instance, Vatankhah and Kouchakzadeh (2008, 2009) improved coefficients from the approximation of Sonnad and Goudar (2006, 2007) in a way that maximum relative error decreases from 0.800% to 0.147%. Their model, which is non-linear, was solved using the Solver toolbox of Microsoft Excel (PRWeb, 2013). This software can solve the full spectrum of optimization problems from simple linear programming to complex models. The software includes two approaches (the evolutionary solver and the generalized reduced gradient solver that uses multi-start methods) to finding global optimal solutions.

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http://dx.doi.org/10.1016/j.petrol.2013.12.001

Table 1
Accurate friction factor, *f*, computed with six significant digits for various values of log(Re), first column, and –log(*ε*/*D*), first row.

	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00
4.00	0.103130	0.077651	0.061236	0.050350	0.043099	0.038349	0.035332	0.033481	0.032378	0.031735	0.031366	0.031156	0.031037	0.030970	0.030932	0.030910	0.030898	0.030892	0.030888	0.030886	0.030884
4.05	0.102955	0.077424	0.060937	0.049958	0.042597	0.037735	0.034620	0.032692	0.031537	0.030861	0.030472	0.030250	0.030124	0.030053	0.030013	0.029990	0.029978	0.029971	0.029967	0.029964	0.029963
4.10	0.102799	0.077222	0.060668	0.049603	0.042139	0.037169	0.033954	0.031949	0.030740	0.030029	0.029619	0.029384	0.029251	0.029176	0.029134	0.029110	0.029096	0.029089	0.029084	0.029082	0.029081
4.15	0.102659	0.077040	0.060427	0.049282	0.041721	0.036646	0.033334	0.031250	0.029985	0.029237	0.028804	0.028556	0.028416	0.028336	0.028291	0.028266	0.028252	0.028243	0.028239	0.028236	0.028235
4.20	0.102534	0.076878	0.060210	0.048992	0.041341	0.036165	0.032757	0.030593	0.029270	0.028484	0.028027	0.027765	0.027616	0.027531	0.027484	0.027457	0.027442	0.027433	0.027429	0.027426	0.027424
4.25	0.102423	0.076732	0.060016	0.048731	0.040995	0.035724	0.032221	0.029977	0.028593	0.027767	0.027285	0.027007	0.026850	0.026760	0.026710	0.026681	0.026665	0.026656	0.026651	0.026648	0.026647
4.30	0.102324	0.076603	0.059842	0.048496	0.040681	0.035319	0.031723	0.029398	0.027953	0.027085	0.026576	0.026283	0.026116	0.026021	0.025967	0.025937	0.025920	0.025910	0.025905	0.025902	0.025900
4.35	0.102235	0.076487	0.059686	0.048285	0.040397	0.034948	0.031263	0.028857	0.027349	0.026437	0.025900	0.025590	0.025412	0.025312	0.025255	0.025223	0.025205	0.025194	0.025189	0.025186	0.025184
4.40	0.102156	0.076383	0.059547	0.048094	0.040140	0.034610	0.030837	0.028351	0.026779	0.025821	0.025255	0.024926	0.024738	0.024631	0.024571	0.024537	0.024518	0.024507	0.024501	0.024497	0.024495
4.45	0.102085	0.076291	0.059422	0.047924	0.039907	0.034301	0.030445	0.027880	0.026242	0.025237	0.024639	0.024291	0.024092	0.023978	0.023914	0.023878	0.023857	0.023846	0.023839	0.023836	0.023834
4.50	0.102022	0.076208	0.059310	0.047770	0.039697	0.034020	0.030084	0.027440	0.025736	0.024682	0.024052	0.023684	0.023472	0.023352	0.023283	0.023245	0.023223	0.023211	0.023204	0.023200	0.023198
4.55	0.101966	0.076134	0.059210	0.047633	0.039508	0.033764	0.029752	0.027031	0.025261	0.024157	0.023492	0.023102	0.022878	0.022750	0.022677	0.022636	0.022613	0.022600	0.022592	0.022588	0.022586
4.60	0.101916	0.076069	0.059121	0.047510	0.039338	0.033532	0.029447	0.026652	0.024815	0.023659	0.022959	0.022546	0.022308	0.022171	0.022094	0.022050	0.022026	0.022012	0.022004	0.021999	0.021997
4.65	0.101872	0.076010	0.059041	0.047399	0.039184	0.033322	0.029168	0.026300	0.024397	0.023188	0.022451	0.022014	0.021761	0.021616	0.021533	0.021487	0.021461	0.021446	0.021437	0.021433	0.021430
4.70	0.101832	0.075957	0.058970	0.047300	0.039046	0.033132	0.028914	0.025975	0.024006	0.022743	0.021967	0.021505	0.021236	0.021082	0.020994	0.020945	0.020917	0.020901	0.020892	0.020887	0.020884
4.75	0.101796	0.075911	0.058906	0.047212	0.038922	0.032960	0.028681	0.025676	0.023640	0.022324	0.021507	0.021019	0.020733	0.020569	0.020476	0.020423	0.020393	0.020376	0.020366	0.020361	0.020358
4.80	0.101765	0.075869	0.058849	0.047133	0.038811	0.032804	0.028470	0.025399	0.023300	0.021928	0.021070	0.020554	0.020251	0.020076	0.019976	0.019920	0.019888	0.019870	0.019860	0.019854	0.019851
4.85	0.101737	0.075832	0.058799	0.047062	0.038711	0.032664	0.028277	0.025146	0.022984	0.021556	0.020656	0.020110	0.019788	0.019602	0.019496	0.019435	0.019401	0.019382	0.019371	0.019365	0.019362
4.90	0.101711	0.075799	0.058753	0.046999	0.038622	0.032538	0.028103	0.024913	0.022690	0.021207	0.020263	0.019686	0.019345	0.019147	0.019033	0.018969	0.018932	0.018912	0.018900	0.018894	0.018890
4.95	0.101689	0.075769	0.058713	0.046942	0.038542	0.032425	0.027944	0.024700	0.022417	0.020879	0.019890	0.019282	0.018920	0.018709	0.018588	0.018519	0.018480	0.018458	0.018446	0.018439	0.018435
5.00	0.101669	0.075743	0.058677	0.046891	0.038470	0.032322	0.027801	0.024505	0.022165	0.020572	0.019538	0.018897	0.018512	0.018288	0.018159	0.018085	0.018044	0.018020	0.018007	0.017999	0.017995
5.05	0.101651	0.075719	0.058645	0.046846	0.038406	0.032231	0.027672	0.024327	0.021933	0.020286	0.019206	0.018530	0.018123	0.017884	0.017746	0.017667	0.017623	0.017597	0.017583	0.017575	0.017571
5.10	0.101635	0.075698	0.058616	0.046806	0.038348	0.032148	0.027555	0.024166	0.021719	0.020019	0.018892	0.018181	0.017749	0.017495	0.017348	0.017264	0.017216	0.017189	0.017174	0.017166	0.017161
5.15	0.101621	0.075679	0.058590	0.046770	0.038297	0.032074	0.027449	0.024019	0.021522	0.019770	0.018597	0.017849	0.017392	0.017122	0.016965	0.016875	0.016824	0.016795	0.016779	0.016770	0.016764
5.20	0.101608	0.075663	0.058568	0.046738	0.038251	0.032008	0.027354	0.023885	0.021342	0.019539	0.018320	0.017535	0.017051	0.016764	0.016596	0.016500	0.016445	0.016414	0.016397	0.016387	0.016381
5.25	0.101597	0.075648	0.058547	0.046709	0.038210	0.031949	0.027269	0.023764	0.021177	0.019326	0.018060	0.017236	0.016726	0.016420	0.016241	0.016138	0.016079	0.016046	0.016028	0.016017	0.016011
5.30	0.101587	0.075635	0.058529	0.046684	0.038173	0.031896	0.027192	0.023655	0.021026	0.019128	0.017816	0.016954	0.016415	0.016090	0.015899	0.015789	0.015726	0.015691	0.015670	0.015659	0.015653
5.35	0.101578	0.075623	0.058513	0.046661	0.038140	0.031848	0.027123	0.023556	0.020888	0.018946	0.017589	0.016688	0.016118	0.015773	0.015570	0.015452	0.015385	0.015347	0.015325	0.015313	0.015306
5.40	0.101570	0.075612	0.058498	0.046640	0.038111	0.031806	0.027061	0.023467	0.020763	0.018778	0.017376	0.016436	0.015836	0.015470	0.015253	0.015127	0.015055	0.015014	0.014991	0.014978	0.014970
5.45	0.101563	0.075603	0.058486	0.046622	0.038085	0.031768	0.027005	0.023386	0.020649	0.018623	0.017179	0.016199	0.015568	0.015179	0.014948	0.014814	0.014736	0.014692	0.014667	0.014653	0.014646
5.50	0.101557	0.075595	0.058474	0.046606	0.038062	0.031734	0.026955	0.023314	0.020546	0.018482	0.016996	0.015977	0.015313	0.014902	0.014655	0.014511	0.014428	0.014381	0.014354	0.014339	0.014331
5.55	0.101551	0.075587	0.058464	0.046592	0.038041	0.031703	0.026911	0.023248	0.020453	0.018353	0.016826	0.015768	0.015072	0.014636	0.014373	0.014220	0.014131	0.014080	0.014051	0.014035	0.014026
5.60	0.101546	0.075580	0.058455	0.046579	0.038022	0.031676	0.026871	0.023190	0.020368	0.018234	0.016669	0.015572	0.014843	0.014382	0.014103	0.013938	0.013843	0.013789	0.013758	0.013741	0.013731
5.65	0.101542	0.075575	0.058447	0.046567	0.038006	0.031652	0.026835	0.023137	0.020291	0.018127	0.016525	0.015390	0.014627	0.014140	0.013843	0.013667	0.013565	0.013507	0.013474	0.013455	0.013444
5.70	0.101538	0.075569	0.058439	0.046557	0.037991	0.031630	0.026803	0.023090	0.020222	0.018029	0.016392	0.015220	0.014423	0.013910	0.013594	0.013406	0.013297	0.013234	0.013199	0.013178	0.013167
5.75	0.101534	0.075565	0.058433	0.046548	0.037978	0.031611	0.026774	0.023047	0.020160	0.017939	0.016269	0.015061	0.014231	0.013691	0.013355	0.013155	0.013038	0.012970	0.012932	0.012910	0.012898
5.80	0.101531	0.075560	0.058427	0.046540	0.037966	0.031594	0.026749	0.023009	0.020104	0.017859	0.016157	0.014914	0.014051	0.013482	0.013127	0.012913	0.012787	0.012715	0.012673	0.012650	0.012637
5.85	0.101528	0.075557	0.058422	0.046532	0.037955	0.031578	0.026726	0.022975	0.020054	0.017785	0.016055	0.014778	0.013881	0.013285	0.012908	0.012680	0.012545	0.012467	0.012423	0.012398	0.012384
5.90	0.101526	0.075553	0.058417	0.046526	0.037946	0.031564	0.026706	0.022945	0.020008	0.017719	0.015961	0.014653	0.013723	0.013098	0.012699	0.012456	0.012312	0.012228	0.012180	0.012153	0.012138
5.95	0.101523	0.075550	0.058413	0.046520	0.037938	0.031552	0.026687	0.022917	0.019968	0.017659	0.015876	0.014537	0.013575	0.012921	0.012500	0.012240	0.012086	0.011997	0.011945	0.011916	0.011900
6.00	0.101521	0.075548	0.058410	0.046515	0.037930	0.031541	0.026671	0.022893	0.019931	0.017606	0.015799	0.014431	0.013438	0.012754	0.012310	0.012034	0.011869	0.011773	0.011717	0.011686	0.011668
6.05	0.101520	0.075545	0.058406	0.046510	0.037924	0.031531	0.026656	0.022871	0.019898	0.017557	0.015728	0.014333	0.013310	0.012597	0.012129	0.011836	0.011659	0.011556	0.011497	0.011463	0.011443
6.10	0.101518	0.075543	0.058403	0.046506	0.037918	0.031523	0.026643	0.022852	0.019869	0.017513	0.015664	0.014243	0.013191	0.012450	0.011957	0.011646	0.011457	0.011347	0.011283	0.011246	0.011225
6.15	0.101517	0.075541	0.058401	0.046503	0.037912	0.031515	0.026632	0.022834	0.019843	0.017474	0.015607	0.014162	0.013081	0.012312	0.011794	0.011464	0.011263	0.011144	0.011075	0.011036	0.011013
6.20	0.101515	0.075540	0.058399	0.046499	0.037908	0.031508	0.026622	0.022819	0.019819	0.017438	0.015554	0.014087	0.012979	0.012182	0.011640	0.011290	0.011076	0.010948	0.010874	0.010832	0.010808
6.25	0.101514	0.075538	0.058397	0.046497	0.037904	0.031502	0.026612	0.022805	0.019798	0.017407	0.015507	0.014019	0.012886	0.012062	0.011494	0.011125	0.010896	0.010759	0.010680	0.010634	0.010608
6.30	0.101513	0.075537	0.058395	0.046494	0.037900	0.031496	0.026604	0.022792	0.019779	0.017378	0.015465	0.013957	0.012800	0.011949	0.011356	0.010967	0.010723	0.010577	0.010491	0.010442	0.010413
6.35	0.101512	0.075536	0.058393	0.046492	0.037896	0.031491	0.026597	0.022781	0.019763	0.017353	0.015426	0.013901	0.012721	0.011845	0.011227	0.010816	0.010557	0.010401	0.010308	0.010255	0.010225
6.40	0.101511	0.075535	0.058392	0.046490	0.037894	0.031487	0.026590	0.022771	0.019748	0.017330	0.015392	0.013850	0.012649	0.011748	0.011106	0.010674	0.010398	0.010231	0.010132	0.010074	0.010042
6.45	0.101511	0.075534	0.058390	0.046488	0.037891	0.031483	0.026584	0.022762	0.019734	0.017309	0.015361	0.013804	0.012584	0.011659	0.010992	0.010538	0.010246	0.010067	0.009961	0.009899	0.009863
6.50	0.101510	0.075533	0.058389	0.046486	0.037888	0.031480	0.026579	0.022755	0.019722	0.017291	0.015333	0.013763	0.012524	0.011576	0.010885	0.010410	0.010101	0.009910	0.009795	0.009729	0.009691
6.55	0.101510	0.075532	0.058388	0.046485	0.037886	0.031476	0.026574	0.022748	0.019711	0.017275	0.015308	0.013725	0.012469	0.011500	0.010786	0.010289	0.009962	0.009758	0.009635	0.009564	0.009522
6.60	0.101509	0.075531	0.058387	0.046483	0.037885	0.031474	0.026570	0.022741	0.019702	0.017260	0.015286	0.013692	0.012419	0.011431	0.010694	0.010175	0.009830	0.009612	0.009481	0.009404	0.009359
6.65	0.101509	0.075531	0.058386	0.046482	0.037883	0.031471	0.026567	0.022736	0.019693	0.017247	0.015266	0.013661	0.012374	0.011367	0.010608	0.010067	0.009704	0.009473	0.009332	0.009249	0.009201
6.70	0.101508	0.075530	0.058386	0.046481	0.037881	0.031469	0.026563	0.022731	0.019686	0.017235	0.015248	0.013634	0.012334	0.011308	0.010529	0.009967	0.009584	0.009339	0.009188	0.009098	0.009047