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Pressure transient analysis with exponential and power law boundary flux

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ABSTRACT

The widely used conceptual model set to study flow phenomena in an oil reservoir or groundwater system has been for several decades the homogeneous cylindrical model with a fully penetrating well located at its centre. For wells that produce at a constant rate in a bounded reservoir the boundary conditions are usually zero flux at the outer boundary. In this work we analyze the pressure response in the presence of flow at the external border, following an exponential or a power law boundary flux. Our solutions could be used for the analysis and interpretation of the pressure response when it is dominated by the boundary effects or discontinuities.

Contents

1. Introduction

A fully penetrating well located at the centre of a cylindrical reservoir producing at a constant rate is the widely used model to study flow phenomena. Despite this simplified geometry, many complications in well-test interpretation arise due to inner and outer boundary effects. For homogeneous reservoirs the pressure response influenced by the outer boundary effects such as leaky faults, no flow boundaries, and constant pressure boundaries has been studied by several authors (Acosta and Ambastha, 1994; Earlougher et al., 1974; Ehlig-Economides and Ramey, 1981; Fuentes-Cruz et al., 2010).

With regard to heterogeneous reservoirs, there are models available in the literature that address similar issues like compartmentalized reservoirs and geochoke with applications in different areas such as extraction of crude oils, underground oil displacement, well drilling, and aquifer contamination. There are models of flow in

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Nomenclature

Field variables

ϕ	porosity, dimensionless
μ	fluid viscosity, M/LT
k	formation permeability, L^2
c_t	total system compressibility, LT^2/R
r_w	wellbore radius, L
r	radial distance, L
R	reservoir drainage radius, L
p	pressure, M/LT^2
p_i	initial reservoir pressure, M/LT^2
q	oil flow rate, L^3/T
t	time, T
h	formation thickness, L

Dimensionless variables

p_D	dimensionless pressure
r_D	dimensionless radial distance
t_D	dimensionless time
R_D	dimensionless reservoir drainage radius
q_{R_D}	dimensionless terminal (endpoint) boundary influx
τ_D	dimensionless starting time for boundary influx
α	scaling exponent (real number)
$H(t_D)$	Heaviside function

porous media that have considered moving boundary to describe the transient flow of power law fluids (Pascal and Pascal, 1985). The variable flow concept has also been studied for non-Newtonian fluids in bounded and homogeneous domain (Ciriello et al., 2013; Ciriello and Di Federico, 2012).

The model presented in this paper can be equally considered as special cases of these compartmentalized reservoirs, the so-called composite reservoirs consisting of two or more concentric regions with a single well at the centre. The composite models have been generalized in different ways, for example, including effects of various trends of mobility and storativity variations to determine the swept volume in thermal wells (Acosta and Ambastha, 1994). Acosta and Ambastha (1994) present a general methodology for analyzing pressure transient test for composite reservoirs including a fractal region between the homogeneous circular region. On the other hand Corbett et al. (2012) presented a new well testing response considering vertical influx to reservoir.

These early studies have not considered that influx into the reservoir is prescribed by the external boundary condition. Doublet and Blasingame studied the numerical pressure response and the transient flow at the well due to the variable flow at the boundary. In this work we propose a new temporal variable condition at the external boundary (power law), numerical and analytical solution is obtained for studying pressure response in reservoir. The new well test responses are compared with the pressure curves considering the exponential flux condition at boundary as proposed by Doublet and Blasingame (1995).

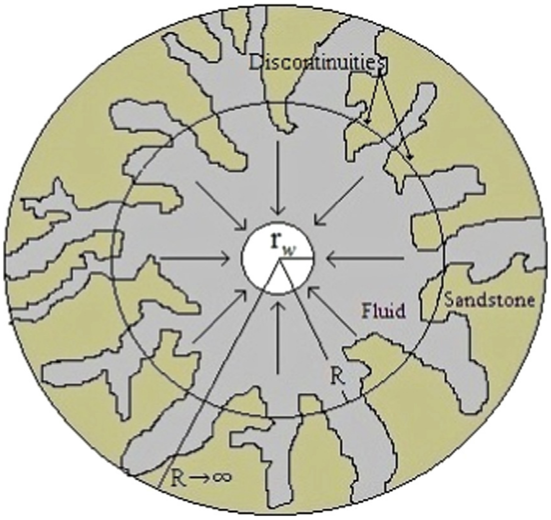

Fig. 1. Agglomerations around the boundary (either shaly sand facies or shale lenses) that can obstruct the flow channels at a radial distance R from the well radius r_w.

Our physical model is a single well centred in a bounded circular reservoir with two different conditions at the inner boundary (constant rate production and constant pressure production) for describing pressure behaviour. Additionally a prescribed flux at the outer boundary is considered. This prescribed flux or variable flow at the boundary means that the influx at the outer boundary is initially zero changing after a time to a fixed value following two different ways: exponential and power law.

It is hoped that by incorporating flux at the boundary we can obtain a better approach to modelling homogeneous reservoirs (mainly on those modelled as closed ones) with some heterogeneity at the external boundary. A closed reservoir is mainly characterized by an impermeable boundary (no-flow condition). Starting from this physical concept, the model analyzed here describes a boundary that provides fluid to the system with a different rate such that the boundary acts like a low-permeability boundary. From geological point of view this can be associated with reservoirs bounded by discontinuities due to agglomerations of shaly sand facies or shale lenses that obstruct partially or totally the flow channels as shown in Fig. 1. Discontinuities due to variations in rock types are not precisely circular, but we assume a uniform distribution around the boundary as an approximation (Rosa, 1996). The rationale or motivation for our model is that in these types of geometries, for example lenses of sand surrounded by shale barriers or fractures, flow inside the lenses occurs in an homogeneous rock but restrictions to flow are located at the boundary where fractures or shale structures induce a subdiffusive flow into the region under exploitation. This scenario is not purely theoretic; it is a plausible model in reservoirs such as Chicontepec (Comisión Nacional de Hidrocarburos, 2010) in Mexico.

In this work we first present the basic theory for a model with variable flux at the boundary. Second we establish the conditions under which each of the models with exponential or power law boundary flux is applicable. Third we present and compare the analytical and numerical approaches of the pressure response for both models. Finally we analyze the characteristic behaviour of the pressure derivative.

2. The basic theory for variable flux at the boundary

One of the basic models for describing fluid flow in cylindrical porous media is described by the diffusion equation in radial coordinates given by

$$\frac{\partial^2 p}{\partial r^2}+\frac{1}{r}\frac{\partial p}{\partial r}=\frac{\phi\mu c_t}{k}\frac{\partial p}{\partial t}, \tag{1}$$