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Well-test response in stochastic permeable media

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ABSTRACT

In this paper, different approaches are employed to study the ramifications of various statistical and geostatistical parameters (i.e. mean and variance, vertical and horizontal correlation lengths and covariance function types) on the well-test results. These approaches include analytical, numerical, and semi-analytical methods, each of which explains the ensemble and/or volume average permeability of heterogeneous reservoirs from transient tests. In particular, in the numerical method, a few thousand pixel-based geostatistical permeability realizations, with various parameters, are generated; and the single-phase flow simulations are performed for each realization. The extracted permeability from the ensemble well-test response is then related to the underlying statistical and geostatistical parameters of permeability distribution. In this paper special attention is paid to a well-test response (i.e. a ramp effect) in a particular heterogeneous reservoir, that is manifested as a steep increase of the well-test derivative curve. A real test example is also presented to give an intuition as to how the permeable heterogeneity is reflected in a real problem. This paper provides the necessary practical insight for constructing a link between the static geological model and the dynamic test data found in highly heterogeneous stochastic reservoirs.

1. Introduction

Well-testing is a standard measurement technique of recording pressure and rate data from a reservoir. The interpretation of the recorded data provides invaluable information about fluid (e.g. condensate drop-out near the wellbore) and reservoir heterogeneities. For common cases of constant production rate, the recorded pressure is translated into some diagnostic plots (e.g. a log–log plot), which facilitate the interpretation and modelling processes. A log–log diagnostic plot consists of two separate curves: Δp (pressure drop) and $\Delta p'$ (pressure derivative) versus time, and for a single-rate drawdown case, they are defined as follows:

$$\Delta p = p_i - p_{wf} \tag{1}$$

$$\Delta p' = \frac{d[\Delta p]}{d[\log(t)]} \tag{2}$$

where, p_i is the initial pressure, p_{wf} is the wellbore pressure (Bourdet, 2002).

Reservoir properties and different heterogeneities show their effects on the derivative curve in terms of distinct slopes and stabilizations. For example, the average well-test permeability is estimated from the radial flow regime, where the derivative curve plateaus off over a definite value. This is an average permeability of a region around the wellbore with a larger scale than the core measurements (Corbett, 2009). The knowledge of scale relationships help scientists and engineers advance towards a better understanding of the heterogeneity distribution and averaging process which assists in finding out how the derived rules from one scale might be applicable to a different scale (Daltaban and Wall, 1998).

In this paper, different approaches are employed to describe the average permeability in the well-test scale. In the analytical (i.e. stochastic) method the permeability is treated as a random function. The effective permeability is estimated from analytical methods using stochastic Darcy's law and the diffusivity equation. This method helps make some direct analytical conclusions which aim to relate the average permeability of the investigated media from the well-test to the geostatistical permeability distribution parameters. However, this method has limitations while applying it in 3-D domains and for higher log-permeability variances. The numerical approach is used as a generalized method to generate the well-test response and extract the effective permeability for the higher variances in the multi-layer cross flow ($k_V \neq 0$) and commingled ($k_V = 0$) reservoirs. In this approach, several 3-D permeability realizations are generated and the single-phase flow simulations using Eclipse 100 simulator (Schlumberger, 2011) are performed to generate the drawdown well-test responses. This interpretation is based on the ensemble transient tests of all flow simulations. The numerical simulation of the detailed geological

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model with an arbitrary permeability distribution provides a unique opportunity to examine the dynamic behaviour of the model and to explore the effect of different input parameters on the outputs of the numerical simulations. On the other hand, the semi-analytical approach considers a single 2-D spatial realization of the permeability and explains the areal averaging process of the well-testing. In this method, not only the effective permeability, but also a permeability-related (i.e. instantaneous permeability) profile can be estimated.

In this paper, special attention is paid to a particular well-test response in commingled reservoirs that is called ramp effect response, where the derivative curve increases by at least one log cycle and eventually plateaus off over the effective permeability-thickness product of the system (Corbett et al., 2012; Hamdi, 2012). The ramp effect is usually associated with an early and a late time plateaus. The existence of the pressure derivative stabilizations benefit the reservoir engineering practice as the average permeabilities can be estimated at each plateau, while the geology footprint is reflected in the transient increase of the derivative curve. The slope and shape of the ramp depend on many geostatistical parameters that will be discussed in detail in the following sections. A real case will be presented to highlight the non-uniqueness in well-test solutions and to show how the knowledge acquired in this paper can be used with a proper interpretation of acquired data in a real problem.

2. Interpretation methods

2.1. Analytical methods

In the analytical approach, the diffusivity equation is treated as a stochastic differential equation and the corresponding parameters have been assumed to be stationary random functions. Then, taking the ensemble average of the flow equation and its boundary conditions one can use the concepts of Green's functions (Carslaw and Jaeger, 1959; Ozkan, 2006) to obtain the effective permeability of the medium under certain conditions. The effective permeability *is* the permeability, which represents the statistically homogeneous medium at a large scale.

In the absence of the extreme sampled permeability values (i.e. zero or infinity), the harmonic average is the lower bound and the arithmetic average is the upper bound of the effective permeability (Cardwell and Parsons, 1945; Dagan, 1989; Renard and de Marsily, 1997; Deutsch, 2002). The effective single-phase permeability of a porous medium is a general function of the flow regimes, geology, and the geostatistical parameters (e.g. variance and the ratios between the length scales characterizing the covariance function $C_{Y=\ln k}$ (Dagan, 1989)), and is therefore difficult to represent by simple mathematical form. However, based on the stochastic approach, and assuming an infinite-domain contains a heterogeneous permeability field, several authors (Dagan, 1993; Paleologos et al., 1996; Tartakovsky et al., 2000; Jankovic et al., 2003; Sanchez-Vila and Tartakovsky, 2007; Gluzman and Sornette, 2008) attempted to obtain the effective permeability of the medium under the steady-state and uniform flow conditions. In all of these studies, the isotropic permeability was assumed to be a stationary random function with a lognormal distribution and finite correlation range. For 1-D and 2-D flow conditions, the generalization of a perturbation approach, the so called Landau–Matheron, also known as the Landau–Lifshitz conjecture (Dagan, 1993; Paleologos et al., 1996), was found to be exact. The conjecture gives the effective permeability of the medium and is written as follows:

$$\frac{k_{eff}}{k_G} = \exp\left[\left(\frac{1}{2}-\frac{1}{D}\right)\sigma_Y^2\right] \tag{3}$$

$$k_G = \exp(m_Y) \tag{4}$$

in which, k_G is the geometric average of permeability, k_{eff} is the effective permeability at distances larger than the correlation length, D is the dimension of space in which flow takes place, and m_Y and σ_Y^2 are the mean and variance of $Y=\ln(k)$. The equation states that the effective permeability is equal to the harmonic average or geometric average under 1-D or 2-D flow conditions respectively. Noetinger (1994) stated that when the number of the flow dimensions increases (i.e. $D\to\infty$) the arithmetic average is obtained. In these situations, the correlation between the pressure gradient and the permeability can be completely disregarded since the flow will preferentially avoid the low permeable regions (Noetinger, 1994). A more comprehensive study was also performed by Noetinger and Gautier (1998). They derived an integro-differential equation for representing the average test response in 2-D heterogeneous reservoirs. They showed that this equation can provide the steady-state asymptote approximation at the late well-test times.

For the 3-D flow condition, the conjecture failed to represent the exact effective permeability demonstrating that there is a dependency on the shape of the covariance function (De Wit, 1995). However, Noetinger (1994) showed that the conjecture holds (but as an approximation), when there is no effect of correlations between different points in the field. He showed that the effective permeability of an uncorrelated isotopic lognormal permeability field in 3-D can be expressed as a power average of the permeabilities: $k_{eff}=\langle k^{1/3}\rangle^3$ (the bracket "$\langle\ldots\rangle$" represents the ensemble averaging operator). This is the same result that was obtained by Desbratas (1992). Based on the Self-Consistent Approximation, Dagan (1993) defined the effective permeability of an isotropic lognormal permeability field in 3-D and 2-D flow conditions without any restriction on the variance. However, later, Dykaar and Kitandis (1992) indicated that the underlying assumptions for Dagan's approach (1993) were not appropriate for the porous medium.

To the best of the author's knowledge, for "time dependent" effective permeability, the published solutions are limited to the cases of mildly heterogeneous permeability fields in infinite domains, in which the lognormal permeability distribution has a small variance compared to unity (Dagan, 1982; Noetinger and Haas, 1996; Tartakovsky and Neuman, 1998; Zhang, 2001). This restriction is a stumbling block on the road to applicability of numerous theoretical analyses to real-world problems (Gluzman and Sornette, 2008). The transient effective permeability obtained in this manner is a complex function of time and space and is dependent on the covariance function of permeability. For the case of infinite domain, isotropic and exponential covariance function, the effective permeability can be formulated in a useful mathematical form that can be obtained by the following equations (Dagan, 1982):

$$\frac{k_{eff}}{k_G} = 1+\sigma_Y^2\left[\frac{1}{2}-\frac{1}{D}+\frac{\beta(t)}{D}\right] \tag{5}$$

where,

$$\beta(t) = \frac{1}{\sigma_Y^2}\int C_Y(|\mathbf{x}-\mathbf{x}'|)G(\mathbf{x},\mathbf{x}',t)d\mathbf{x}' \tag{6}$$

in which, $C_Y(|\mathbf{x}-\mathbf{x}'|)$ is the covariance function, t is time, k_G is geometric average of permeability, D is dimension of flow and $G(\mathbf{x}, \mathbf{x}', \mathbf{t})$ is the transient Green's function. Using an isotropic covariance function in an infinite domain, two useful conclusions can be