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Numerical solution of the nonlinear diffusivity equation in heterogeneous reservoirs with wellbore phase redistribution

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ABSTRACT

We consider the application of the Finite Element Method (FEM) for numerical pressure transient analysis under conditions where no reliable analytical solution is available. Pressure transient analysis is normally based on various analytical solutions of the *linear* one-dimensional diffusion equation under restrictive assumptions about the formation and its boundaries. For example, the formation is either assumed isotropic or a restrictive *a priori* assumption is made about its heterogeneity. The wellbore storage effect is also often considered without regard to the possibility of phase redistribution. In many practical situations such restrictions are not justified and analytical solutions do not exist. Here we present a numerical solution of the nonlinear diffusion equation based on the FEM that can be used without any restrictive *a priori* assumptions. Through the use of the weak formulation of the FEM, solution can be obtained for a heterogeneous medium with discontinuous or nonlinear properties. The weak formulation also enables the handling of time dependent boundary conditions and hence problems involving wellbore storage with significant phase redistribution. The speed and accuracy of the numerical technique is first confirmed by comparison with simple test cases that admit an analytical solution. The practical utility of the proposed method is then demonstrated for a number of test cases that involve discontinuous and nonlinear formation properties and/or wellbore storage with phase redistribution.

1. Introduction

The pressure transient response of the reservoir is a widely used method for reservoir characterization. However, the early time portion of such data sometimes is infected by wellbore effects. To discriminate the reservoir portion from the wellbore portion and extracting a true reservoir parameter, a wellbore model is required. Currently, there are three types of flow models which can be used for modeling fluid flow in the wellbore; empirical correlations, homogeneous models and mechanistic models (Shi et al., 2005a). Empirical correlations are based on matching of experimental data and their range of applicability are generally limited to the range of variables and the particular geometry used in the experiments (Duns and Ros, 1963; Hagedorn and Brown, 1965; Orkiszewski, 1967; Aziz et al., 1972; Beggs and Brill, 1973; Ansari et al., 1994). Homogeneous models that sometimes are called drift-flux models, assume that a single-phase fluid flows in the wellbore and the properties of this fluid is represented by a mixture of properties (Shi et al., 2005a, 2005b). Both empirical correlations and homogeneous models have been developed mainly for steady state multiphase flow in the wellbore. However, during the well test, the wellbore condition is generally kept in single-phase flow and pressure is always changing by time. Therefore, such wellbore flow models are no longer applicable for pressure transient analysis. Mechanistic models, however, are based on the fundamental conservation laws of mass and momentum transfer e.g. Navier–Stokes differential equations. Although, they are accurate and reliable models and can be used in transient condition, the computational burden is significant to solve them and to get reliable solutions. They often encounter convergence problems and the computation time is remarkably long. For instance, initializing Navier–Stokes equations and establishing velocity field in the well, sometimes takes a longer time than solving the diffusion equation in the reservoir. The computation time becomes noticeable especially when the coupled wellbore-reservoir model should be executed for many times to estimate reservoir parameters (Pourafshary et al., 2009; Khadivi et al., 2013).

An alternative approach, which is often used for pressure transient analysis, is considering the well as a boundary condition instead of a separate model. The accuracy of this method in representing the wellbore dynamic depends on the equation that is used as the boundary condition. This type of well modeling is

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Nomenclature

C_D	dimensionless wellbore storage coefficient, $C_D = C_s(1/2\pi\phi c_t h r_w^2)$
$C_{\phi D}$	dimensionless phase redistribution pressure parameter, $C_{\phi D} = C_\phi(kh/qB_o\mu)$
c_ϕ	rock compressibility, Lt^2/m, 1/Pa
c_f	fluid compressibility, Lt^2/m, 1/Pa
c_t	total compressibility, $c_t = c_f + c_\phi$, Lt^2/m, 1/Pa
C_s	wellbore storage coefficient, L^{4}t^2/m, m^3/Pa
D	unit vector in gravity direction, L, m
g	the acceleration of gravity, L/t^2, m/s^2
h	reservoir thickness, L, m
k	reservoir permeability, L^2, m^2
$\bar{\bar{k}}$	symmetric and positive definite 3D permeability tensor
p	reservoir pressure, m/Lt^2, Pa
p_w	wellbore pressure, m/Lt^2, Pa
p_D	dimensionless pressure
$p_{\phi D}$	dimensionless phase redistribution pressure, $p_{\phi D} = p_\phi(kh/qB_o\mu)$
q	flow rate, L^3/t, m^3/s
r	radius L, m
r_D	dimensionless radius, $r_D = (r/r_D)$
r_e	external radius of drainage area L, m
r_w	wellbore radius L, m
S	skin factor
t	time, t, s
t_D	dimensionless time, $t_D = t(k/)\phi\mu c_t r_w^2$
$\vec{u}$	Darcy velocity (superficial) vector, L/t, m/s

Greek letters

ϕ	porosity, %
μ	viscosity, m/Lt, Pa s
ρ_f	fluid density, m/L^3, kg/m^3
τ	phase redistribution time parameter, s
τ_D	dimensionless phase redistribution time, $\tau_D = \tau(k/\phi\mu c_t r_w^2)$

Conversion factors

bbl	× 1.589873	E–01 = m^3
cp	× 1.0	E – 03 = Pa s
ft	× 3.048	E–01 = m
psi	× 6.894757	E + 3 = Pa
mD	× 9.86927574528	E – 16 = m^2

fast and reliable to represent wellbore dynamics in various well conditions. The overall goal of this work is developing an efficient and fast numerical procedure to handle reservoir heterogeneity in the presence of wellbore effects.

2. Problem formulation

In this section, we briefly review the governing flow equations in the reservoir and give the associated weak form.

2.1. Governing equations

Most of the theoretical treatments of pressure transient analysis in well testing consider a well situated in a porous medium of infinite radial extent and assume that the fluids flow to a central cylinder (the well) that is normal to two parallel, impermeable planar barriers. The theoretical analysis is based on simplified solutions of the continuity equation in the porous medium:

$$\frac{\partial}{\partial t}(\rho\phi) + \nabla.(\rho\vec{u}) = 0 \tag{1}$$

where ρ is the density of the formation fluid, ϕ is porosity, $\vec{u}$ is the fluid velocity vector and t is time. Flow of the fluid in the porous media is assumed to follow the form of Darcy's law,

$$\vec{u} = -\frac{\bar{\bar{k}}}{\mu}\nabla(p + \rho g \nabla D) \tag{2}$$

here, $\vec{u}$ is the superficial Darcy velocity vector, p is the reservoir pressure, μ and ρ are the fluid viscosity and density, g is the magnitude of the acceleration due to gravity, ∇D is a unit vector in the direction over which gravity acts and $\bar{\bar{k}}$ is the symmetric and positive definite 3D permeability tensor. Substituting Eq. (2) into the continuity equation yields,

$$\frac{\partial}{\partial t}(\rho\phi) - \nabla.\left(\rho\frac{\bar{\bar{k}}}{\mu}\nabla(p + \rho g \nabla D)\right) = 0 \tag{3}$$

Taking the fluid and rock compressibility as constants, $c_f = (1/\rho)(\partial\rho/\partial p) = \text{constant}$ and $c_\phi = -(1/\phi)(\partial\phi/\partial p) = \text{constant}$, leads to a diffusion equation that describes the temporal and spatial pressure changes in the 3D reservoir,

$$\rho\phi c_t \frac{\partial p}{\partial t} - \nabla.\left(\rho\frac{\bar{\bar{k}}}{\mu}\nabla(p + \rho g \nabla D)\right) = 0, \tag{4}$$

where $c_t = c_f + c_\phi$ is the total compressibility.

It is assumed that the top and bottom surfaces of the reservoir layer are sealed and a uniform (either constant pressure or no flow) boundary condition is imposed at the outer limit of the reservoir. It is further assumed that there is no variation in θ direction (radial symmetry) and the reservoir is long enough to be assumed as a single thin layer reservoir. Then with such boundary conditions, the problem geometrically and physically is a one-dimensional symmetric problem.

$$\phi c_t h \frac{\partial p}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left[\frac{kh}{\mu} r\left(\frac{\partial p}{\partial r}\right)\right] = 0 \tag{5}$$

The analysis of the measured pressure transient is based on the solution of Eq. (5) subject to suitable boundary conditions. In the case of an isotropic reservoir the following initial and boundary conditions are often adopted. As the initial condition, the pressure in the reservoir is assumed to be uniform at a given initial value, $p(r, t=0) = p_i$. For the outer boundary condition, the pressure gradient at the outer limit of the reservoir is zero at all times, $(\partial p(r,t)/\partial r) = 0$ at r_e. This may mean that the pressure transient will not reach to the end of the reservoir, the so-called infinite acting reservoir. Alternatively, for long durations or small radius reservoirs it may mean that the reservoir is closed at its outer limit, $\vec{\boldsymbol{n}}.\nabla\vec{u} = 0$ $(r = r_e)$.

Depending on the assumptions made regarding behavior within the wellbore, three major alternatives are used:

Negligible wellbore storage: accumulation of fluid within the wellbore is ignored all together and the fluid issuing from the reservoir is withdrawn directly into the wellbore, the so-called line