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Fractional diffusion in rocks produced by horizontal wells with multiple, transverse hydraulic fractures of finite conductivity

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ABSTRACT

Assuming diffusion in the rocks to be anomalous, a flux law that is nonlocal in space and time is used to develop a mathematical model for fractured rocks drained by a horizontal well produced through multiple transverse hydraulic fractures. As a result the governing differential equation is fractional in character. The conductivity of the fractures is assumed to be finite and their properties (width, length, permeability, etc.) may be variable. Expressions for the well response that produces at a constant rate or at a constant pressure are derived in terms of the Laplace transformation. Approximate analytical solutions are derived and the analytical development provides perspectives on short and long-time well behaviors. In addition to outlining characteristic features of the model, the analytical solutions are useful in verifying numerical computations. The computational results obtained by the Stehfest algorithm establish the robustness and viability of the mathematical model. Comparisons with classical diffusion are noted.

1. Introduction

Production of naturally fractured rocks by a horizontal well through one or more transverse, hydraulic vertical fractures is considered. As we consider flow in disordered structures with a complex geology, a flux law that is an asymptotic approximation of the CTRW process that is nonlocal in time and space is used (Gorenflo et al., 2002; Paradisi et al., 2001). As a result, the transient-flow equation contains fractional derivatives. Fractional derivatives are the consequence of the long-time, long space limit of CTRW models. This view postulates that diffusion is anomalous. Other forms of convolved flux expressions such as those noted by Nigmatullin (1984, 1986) who assumes reservoir-rock properties to be fractals also lead to fractional derivatives and may be incorporated in the model considered here. A discussion of fractional operators may be found in works by Oldham and Spanier (1974), Miller and Ross (1993), Samko et al. (1993), Podlubny (1998), Saxena et al. (2006), Hilfer (2008) and Herrmann (2011) to cite only a few sources.

As is well known three options are available for modeling fractured rocks: continuum, discrete or hybrid schemes. From a number of perspectives the continuum approach is adequate and provides an accurate tool for evaluating well performance. The conductivity of the hydraulic fractures intercepting the wellbore is assumed to be finite, and the properties of each fracture such as length, width, and permeability may be different. Solutions obtained in this study are compared with the classical diffusion reported in Raghavan et al. (1997), and the consequences of using the convolved flux law are demonstrated. That work has been an invaluable resource over the years. But it is difficult to fully explain the many trends in production where shear fractures, micro-fractures in the rock matrix and the like play a dominant role. Exploring results from models of the kind proposed here may lead to additional insights and the understanding of the performance of such systems.

2. Anomalous diffusion: principal considerations

Studies of anomalous diffusion in fractured rocks where the geometry is complex as a result of fractures and pores are based on the fact that transients that govern fluid movement in such structures are defined by a space–time behavior where the second moment of the transient is of the form

$$\langle r^2 \rangle \sim t^a, \tag{2.1}$$

Where, r is the distance, t is the time, the exponent, a, is a constant that is less than 1, and the symbol $\langle\ \rangle$ represents the second moment. Early works such as Chang and Yortsos (1990) considered models based on fractal structures using formulations outlined in O'Shaughnessy and Procaccia (1985a,b) based on the concept that

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Nomenclature

a	arbitrary constant
B	formation volume factor (L^3/L^3)
c	compressibility ($L\,T^2/M$)
D	distance between the outer fractures (L)
d	Euclidean dimension
d_f	fractal (Hausdorff) dimension
d_w	anomalous diffusion coefficient (random walk dimension)
h	thickness (L)
$K_\nu(z)$	modified Bessel function of order ν
k	permeability (L^2)
k_f	permeability of hydraulic fracture (L^2)
k_α	see (2.6)
L_f	half-length of hydraulic fracture (L)
ℓ	reference length (L)
n	number of fractures
p	pressure ($M/L/T^2$)
p'	logarithmic derivative ($M/L/T^2$)
q	rate (L^3/T)
t	time (T)
$t^\star$	time function (see 4.6) ($T^{1-\alpha}$)
w	width (L)
α	exponent
$\Gamma(x)$	gamma function
γ	Euler's constant (0.5772…)
$\Sigma(n, D, L_f)$	pseudoskin factor; multiple-fracture system
σ	pseudoskin factor; single fracture
η	diffusivity; various
$\tilde{\eta}_i$	diffusivity; see (3.18)
λ	mobility; various
μ	viscosity ($M/L/T$)
ϕ	porosity (L^3/L^3)
ϕ_f	porosity of hydraulic fracture (L^3/L^3)
$\psi(x)$	Digamma function

Subscripts

D	dimensionless
i	coordinate, initial
w	well bore

Superscript

–	Laplace transform

the diffusivity, η, that governs transients is of the form (Gefen et al., 1983)

$$\eta \propto r^{-\theta}, \tag{2.2}$$

where $\theta > 0$, and $a = 2/d_w$, where d_w $(\theta + 2)$ is the random walk dimension (anomalous diffusion coefficient). As may be expected, (2.2) leads to power-law forms for both permeability and porosity in terms of the fractal (Hausdorff) dimension, d_f, random walk dimension, d_w, and the Euclidean dimension, d. Thus the flux law for the system is given by

$$q(r,t) = \frac{k(r)}{\mu}\frac{\partial p(r,t)}{\partial r}, \tag{2.3}$$

with the permeability $k(r) = k_c r^{d_f - d - d_w + 2}$ showing an explicit power-law dependence in space where k_c is a system constant. The asymptotic form of the probability density function (instantaneous source function) that is obtained as a result is of the form

$$p(r,t) \sim t^{-d_f/d_w} \exp\left[-c\left(\frac{r}{t^{1/d_w}}\right)^{d_w}\right], \tag{2.4}$$

and the solution given by them for the pressure drop for a well producing at a constant rate in a system of infinite extent may be expressed in the form of a *complementary incomplete gamma function*: $\Gamma(a,x) = \int_x^\infty \zeta^{a-1}\exp(-\zeta)\,d\zeta$. This solution leads to the conclusion that the pressure drop, Δp, at long enough times is of the form $\Delta p \sim t^\alpha$, where α is a constant unlike the conventional expression $\Delta p \sim \ln t$. Many researchers have reported that the flow dimensions in fractured rocks may be different from 2 if we were to assume classical diffusion (d_w=2); see, for example, Bangoy et al. (1992), Hamm and Bidaux (1996), Leveinen et al. (1998) and Riemann et al. (2002). Further, studies such as Le Borgne et al. (2004, 2007) report a flow dimension in the range 1.3–1.7 for many interference tests. By considering responses at all observation wells simultaneously, they obtain a value of 2.8 for d_w. Signatures of the pressure-derivative curves in field tests shown in Acuna et al. (1995) and Flamenco-Lopez and Camacho-Velazquez (2001) do provide convincing evidence for models based on transient flow in fractal structures. Attempts to understand the movement of solutes in fractured rocks also suggest the need to go beyond classical diffusion; see Haggerty et al. (2000), Becker and Shapiro (2000) and Reimus et al. (2003).

Recognizing that the basic model in O'Shaughnessy and Procaccia (1985a,b) is asymptotic in nature, many other forms of the probability density function have been proposed and explored as in Giona and Roman (1992) and Metzler et al. (1994), although Schulzky et al. (2000) indicate that not all of them work for all time ranges. One such expression of the form

$$p(r,t) \sim t^{-d_f/d_w} \exp\left[-c\left(\frac{r}{t^{1/d_w}}\right)^{d_w/(d_w-1)}\right], \tag{2.5}$$

is frequently noted. Such formulations lead to differential equations containing fractional derivatives and suggest that a flux law different from that in (2.3) is required. One form that provides for this option is

$$\mathbf{q}(\boldsymbol{x},t) = -\lambda_\alpha \frac{\partial^{1-\alpha}}{\partial t^{1-\alpha}}[\nabla p(\boldsymbol{x},t)], \tag{2.6}$$

where $\lambda_\alpha = k_\alpha/\mu$ and $\alpha < 1$, and $\partial^\alpha f(t)/\partial t^\alpha$ is the fractional derivative defined in the Caputo (1967) sense:

$$\frac{\partial^\alpha}{\partial t^\alpha}f(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t dt'(t-t')^{-\alpha}\frac{\partial}{\partial t'}f(t'), \tag{2.7}$$

where $\Gamma(x)$ is the Gamma function. Expressions such as (2.6) lead to a differential equation that contains fractional derivatives. Such an equation would represent the long time and/or large distance asymptotic limit of the CTRW process. The advantages of using fractional derivatives may be found in studies such as Benson et al. (2000, 2001) to cite only a couple of examples. We may also show that the exponent, α, plays a role analogous to d_w, and that

$$d_w = \frac{2}{\alpha}. \tag{2.8}$$

There is one other advantage for the immediate problem of interest to us, fractured wells. Extending the observations inherent in (2.3) and (2.4) is difficult as shown in Beier (1994) for that we need to consider flow in a system other than a cylindrical geometry; see Raghavan and Chen (in press). Beier (1994) notes that difficulties