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Thermal adaptive implicit method: Time step selection

J. Maes [a,]*, A. Moncorgé [a], H. Tchelepi [b]

[a] Total CSTJF, Avenue Larribau, 64000 Pau, France
[b] Green Earth Sci. 96, 397 Panama Mall, Mitchell Building 101, CA 94305-2210 Stanford, USA

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ABSTRACT

We present new linear-stability criteria for the Thermal Adaptive Implicit Method (TAIM). The analysis is applied to the mass and energy conservation equations that describe the flow and transport of an arbitrary number of components, which can partition across multiple fluid phases, in the presence of thermal effects. The existing TAIM criteria (Moncorgé and Tchelepi, 2009) do not guarantee oscillation-free numerical solutions for thermal compositional displacement processes that involve very steep temperature fronts. We derive a new stability limit on temperature that overcomes these numerical problems. The methodology is based on linear-stability analysis of the standard low-order space and time discretization schemes of the conservation laws used in general-purpose thermal-compositional reservoir simulators. Specifically, for spatial discretization, phase-based, upstream weighting is used for first derivatives and central differencing is used for second derivatives.

In terms of the robustness and accuracy of the TAIM stability limits, our analysis and computational results indicate that honoring the divergence of the total-velocity in the linearized system of coupled mass and energy conservation equations is more important than accounting for the rock and fluids compressibility effects. Moreover, we demonstrate through scaling analysis and numerical examples that for most problems of practical interest, a simple stability criterion obtained by assuming incompressible multiphase flow is quite robust. The relationship between the full and simplified stability criteria is analyzed in detail. The methodology is demonstrated using several thermal–compositional examples, including Steam Assisted Gravity Drainage (SAGD).

Finally, the criterion for the numerical stability of the temperature is divided into convection and conduction parts. Detailed testing using several simulation models shows clearly that the conduction part of the criterion is quite important across the parameter space of practical interest. Thus, in order to simulate the flow dynamics of large, thermal–compositional reservoir models, the conduction term should always be discretized implicitly, and the TAIM stability criteria should be applied to the mass conservation equations and the convection terms in the energy balance. This means that temperature, like pressure, is an unknown variable in every gridlock.

1. Introduction

Thermal processes to recover heavy oil (high viscosity, low API) from subsurface reservoirs involve complex nonlinear dynamics. The multiphase flow, multi-component transport, and mass transfer of components across multiple fluid phases as a function of pressure and temperature are described by strongly coupled nonlinear conservation equations and constraint relations (Coats, 1980).

Reservoir flow simulators usually employ a finite-volume method to discretize the mass and energy conservation equations. Single-point, phase-based upstream weighting is used for spatial first derivatives, and centered differencing is used for second derivatives in space. For time discretization, the Fully Implicit Method (FIM) is widely used. In order to solve the resulting discrete system of equations associated with FIM, the Newton iterative method is used. Each FIM Newton iteration involves constructing the full Jacobian matrix (linearization), solving the coupled linear system of equations for the primary implicit variables, and updating the approximate solution. The major attraction of FIM is that it is unconditionally stable, so that – in theory – there are no restrictions on the size of the allowable time step. In practice, however, the time step is chosen based on accuracy and efficiency considerations. The major disadvantage of FIM is that building and solving FIM Jacobian matrices for every Newton iteration of every time step is computationally expensive. This is especially the case for highly detailed field-scale reservoir models involving large numbers of components.

The use of mixed-implicit formulations, in which some variables can be treated explicitly in time, can reduce the computational cost on a per-Newton basis. Examples of mixed-implicit

* Corresponding author. Tel.: +44 7582631432.
E-mail addresses: jul.maes@laposte.net, julien.maes@total.com (J. Maes), arthur.moncorge@total.com (A. Moncorgé), tchelepi@stanford.edu (H. Tchelepi).

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Nomenclature

α_e	conversion factor Psi → Btu/ft^3
P	pressure
T	temperature
S_p	saturation of phase p
x_c	is the mole fraction of component c in oil.
y_c	is the mole fraction of component c in gas.
ρ_p	molar density of phase p
H_p	molar enthalpy of phase p
λ_t	total mobility of the fluids
u_t and u_p	total-velocity and velocity of phase p
ϕ	porosity
K	absolute permeability
$\rho_r U_r$	rock internal energy
κ	thermal conductivity of the fluid and the rock
V	bulk volume of a grid block
Δx	space discretization
Δt	time discretization
$q_p = (V/\Delta x)u_p$	volumetric flux of phase p
$T_h = (V/\Delta x^2)\kappa$	heat transmissivity
$c_t = \sum_p (1/\rho_p)(\partial\rho_p/\partial P)S_p$	
$d_t = \sum_p (1/\rho_p)(\partial\rho_p/\partial T)S_p$	
$c_f u_t = \sum_p (1/\rho_p)(\partial\rho_p/\partial P)u_p$	
$d_f u_t = \sum_p (1/\rho_p)(\partial\rho_p/\partial T)u_p$	
$\gamma_P = \sum_p \rho_p(\partial H_p/\partial P)S_p - \alpha_e$	
$\gamma_T = \sum_p \rho_p(\partial H_p/\partial T)S_p + ((1-\phi)/\phi)\partial\rho_r U_r/\partial T$	
$e_P u_t = \sum_p \rho_p(\partial H_p/\partial P)u_p$	
$e_T u_t = \sum_p \rho_p(\partial H_p/\partial T)u_p$	
L	characteristic length of the reservoir
$\mathbb{P} = u_t L/\lambda_t K$	characteristic pressure
$\mathbb{T} = \lvert\gamma_P\rvert\mathbb{P}/\gamma_T$	characteristic temperature
x_D, t_D, P_D, T_D	dimensionless space, time, pressure and temperature

formulations for reservoir simulation include IMPES (IMplicit Pressure, Explicit Saturations) (Coats, 2000) and IMPSAT (IMplicit Pressure and Saturations, explicit treatment of the other variables) (Cao and Aziz, 2002; Quandalle and Savary, 1989). For the same time step size, mixed-implicit formulations are computationally more efficient and also more accurate (less numerical dispersion) than FIM. However, using explicit time discretization for some variables (e.g., saturations in IMPES, compositions in IMPSAT) leads to restrictions on the allowable time step size (Cao and Aziz, 2002; Coats, 2003). In finely gridded, heterogeneous reservoir models, the restrictions on the time step size associated with IMPES, for example, can be quite severe.

In most settings, the time step restriction associated with explicit schemes is usually linked to a small number of computational cells (control volumes) in the reservoir model. This has led to the development of the Adaptive Implicit Method (AIM) (Thomas and Thurnau, 1983; Russell, 1989), which treats some unknowns implicitly and others explicitly. In AIM, implicit treatment is reserved for the variables that really need it and the implicit/explicit treatment (labeling) is dynamic in space and time. For example, at a particular time and for the selected time step size, a variable (e.g., water-saturation) can be treated implicitly in certain cells and explicitly in the other cells. Moreover, for a given variable, the decomposition into implicit and explicit sets for a given time step can be completely different from the previous time step.

The switching (or selection) criteria used to label a discrete variable implicit or explicit are usually based on linear-stability analysis (Russell, 1989). Having accurate switching criteria that are based on sharp stability limits is necessary in order to develop a solution strategy that is both robust and computationally efficient. The stability criteria are usually derived by linearizing the governing equations and using the von Neumann method based on a Fourier series decomposition. The first complete statement of the stability limits for isothermal compositional displacements was presented by Coats (2003). His work paved the way for making AIM a preferred formulation for general-purpose compositional reservoir simulation.

Recently, Moncorgé and Tchelepi (2009) described a Thermal AIM (TAIM) strategy to model compositional displacements involving heat transfer. This was motivated by the desire to improve the efficiency of modeling thermal recovery processes, such as steam flooding and Steam Assisted Gravity Drainage (SAGD), when large numbers of components are needed to describe the complex fluid system. They performed linear-stability analysis of the coupled thermal–compositional system of equations and they derived the stability limits, including a criterion for temperature. Together with the isothermal compositional criteria derived by Coats (2003), the full set of stability criteria was used to perform TAIM simulations for several test cases. Although the temperature criterion presented by Moncorgé and Tchelepi (2009) is satisfactory for most settings of practical interest, they pointed out that the assumption of a divergence-free total-velocity may be inadequate in the neighborhood of temperature fronts. Here, we isolate the shortcomings of the previous temperature stability limit. We then remove some of the assumptions used to derive the coupled overall mass and energy balances and we obtain a new stability limit for temperature. The improved criterion has proved to be quite effective for time step selection.

The paper is organized as follows. In the next section, we summarize the previous TAIM stability criteria (Moncorgé and Tchelepi, 2009) and we show that for steep temperature fronts, their stability limit does not guarantee oscillation-free temperature profiles. We then formulate a coupled pressure–temperature system that relaxes some of the previous assumptions and we derive an improved temperature stability limit. We then obtain a simpler, yet robust, criterion by assuming incompressible multiphase flow. Finally, we present a TAIM scheme with implicit treatment of conduction. The effectiveness of the proposed modifications to the TAIM framework is demonstrated using several test cases.

2. Stability criteria: previous findings

We summarize the recent stability analysis of thermal–compositional displacements (Moncorgé and Tchelepi, 2009). These large-scale processes involve multiple components that can partition across multiple fluid phases as a function of temperature, pressure, and composition. We account for mass and energy transfer in the presence of mass and thermal convection and thermal conduction. We assume that, regarding stability with respect to pressure and temperature, the system can be considered as a multiphase flow without mass exchange across phases. We also neglect capillarity in the accumulation term. Finally, rock is assumed incompressible in all the reservoir.

The overall mass balance can be written as

$$\phi c_t \frac{\partial P}{\partial t} + \phi d_t \frac{\partial T}{\partial t} = -c_f u_t \cdot \nabla P - d_f u_t \cdot \nabla T - \nabla \cdot u_t, \quad (1)$$