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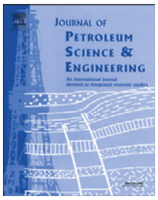

Universal scaling of spontaneous imbibition for arbitrary petrophysical properties: Water-wet and mixed-wet states and Handy's conjecture

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ABSTRACT

Spontaneous imbibition (SI) is a key process in many petrophysical applications, ranging from the mass transfer in fractured reservoirs during a waterflood to wettability characterization of rock samples, or steam migration in geothermal reservoirs. Scaling groups are an essential tool for upscaling laboratory data and modeling and describing SI. A general form has been debated for over 90 years, and several dozen specific groups have been proposed. Here, we give the first general scaling group for arbitrary wettability state, viscosity ratios, rock type, initial water content, and boundary conditions. The result is obtained by extending recent findings for water-wet systems but otherwise arbitrary properties (Schmid and Geiger, 2012) to the mixed-wet case. The group is based on the only known exact, general solution to Darcy's equation with capillarity, and we show that this solution can be viewed as the capillary analogue to the Buckley–Leverett solution for viscous dominated flow. Our group serves as a 'master equation' that contains many of the previously obtained groups as special cases, and its generality can be used to give the first predictive theory for the validity range of specific groups. Based on the universal group, we show that SI is best characterized by the cumulative inflow of the wetting phase and not by the movement of the wetting front, as has been conjectured. Furthermore, our results give strong evidence that Darcy's equation is suitable for describing SI, contrary to what has been hypothesized. The general correlation can be fitted by an exponential model for mass transfer that closely correlates 45 published water–oil, and water–air SI experiments obtained for widely different petrophysical properties.

1. Introduction

Spontaneous imbibition (SI) occurs if a wetting fluid (like water or brine) enters a porous medium, and displaces a non-wetting fluid (like oil, gas or CO_2), driven by capillary forces only. It is a process that is of crucial importance for the evaluation of the wettability of a rock (Jadhunandan and Morrow, 1991; Morrow et al., 1994), and also is the key production mechanism in the world's largest remaining oil reservoirs (Morrow and Mason, 2001). Over 60% of the world's remaining oil reserves are stored in naturally fractured carbonate rocks (Beydoun, 1998). There, the oil is locked in the low permeability rock matrix, surrounded by high permeability fractures, and SI of water is often the only way by which the oil migrates from the rock matrix into the fractures and be produced.

Furthermore, SI is important for the trapping of CO_2 in coal seam and the creation of methane (Chaturvedi et al., 2009), steam migration in geothermal reservoirs (Li and Horne, 2009) and the mechanical stability and distribution of gas hydrates (Anderson et al., 2009; Clennell et al., 1999).

Scaling groups are used to characterize the influence of key parameters on SI, and are essential in any context where SI needs to be understood. For example, they are the bottleneck for an appropriate upscaling of laboratory data (Morrow and Mason, 2001), lie at the heart of modeling and simulating flow in fractured and heterogeneous reservoirs (Barenblatt et al., 1960; Warren and Root, 1963), or are needed as the starting point for evaluating the feasibility of water injection into geothermal reservoirs (Li and Horne, 2009). The enormous practical importance of SI and scaling groups has led to major research activity in that field. In order to resolve how key parameters influence SI and how they should be incorporated into specific scaling groups, a great number of numerical studies on the continuum scale (Behbahani et al., 2006; Delijani and Pishvaie, 2010; Hazlett, 1995; Pooladi-Darvish and Firoozabadi, 2000; Ruth et al., 2000; Standnes, 2006), the pore-scale (Behbahani and Blunt, 2005; Gruener et al., 2012), and the molecular scale (Martic et al.,

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Nomenclature

Greek symbols and units

λ	similarity variable ($\mathrm{mt}^{-1/2}$)
λ_α	mobility of phase α (s/Pa)
μ_α	viscosity of phase α (Pa/s)
$\Delta\rho$	density difference between water and oil phase ($\mathrm{kg/m^3}$)
σ	surface tension (N/m)
$\tau(S_w)$	coefficient for dynamic p_c (Pa s)
τ_c	characteristic time (1/s)
ϕ	porosity (–)

Roman symbols and units

A	measurement of a porous medium's, ability to imbibe, Eq. (8) ($\mathrm{ms}^{-1/2}$)
A_i	area open to imbibition (m^2)
N_B^{-1}	inverse Bond number (–)
c	proportionality constant (–)
$D(S_w)$	capillary dispersion, Eq. (6) ($\mathrm{m^2/s}$)
f	fractional flow function without p_c, Eq. (6) (–)
f_i	abbreviation for $f(S_{wi})$ (–)
F	fractional flow function with p_c Eq. (11) (–)
g	gravitational constant ($\mathrm{m/s^2}$)
H	height of core (m)
J	Leverett J-function (–)
K	absolute permeability (m^2)
L_c	characteristic length, Eq. (18) (m)
l_{A_i}	distance between A_i and now-flow boundary (m)
p_α	pressure (Pa)
p_d	entry pressure (Pa)
$Q_w(t)$	cumulative 1D volume of wetting phase injected/imbibed (m)
q_t	total velocity (m/s)
R	recovery (%)
R_∞	ultimate recovery (%)
S	saturation (–)
S_w^*	effective saturation (–)
S_{wi}	initial water saturation (–)
S_{BC}	water saturation at left boundary (–)
$S_{\alpha r}$	residual saturation of the phase α (–)
t	time (s)
t_a	ageing time (s)
t^*	time when the analytical solutions stop to be valid (s)
$t_{d,inflow}$	dimensionless time (s)
$t_{d,front}$	dimensionless time (s)
$t_{d,Ma}$	dimensionless time (s)
V_b	the bulk volume of the matrix (m^3)

Subscripts

α	$\alpha \in \{w,o\}$
c	capillary
m	matrix
f	fractures
S	saturation
SI	spontaneous imbibition
T	transversal
w	water
o	oil
WF	water flood

2002), experiments (see Tables 3–8), and analytical solutions for special cases of SI (Table 1) have been proposed (Table 2). Despite this intense research activity, however, and although the research on scaling groups and SI spans more than 90 years (Lucas, 1918; Washburn, 1921), not even apparently simple questions – like the influence of viscosity ratios on SI – have been resolved satisfactorily (Schmid and Geiger, 2012).

In this work, we extend a universal scaling group for water-wet systems (Schmid and Geiger, 2012) to the case of mixed-wet systems, and give the first universal scaling group for spontaneous, counter-current imbibition for arbitrary petrophysical properties. We show the validity of our group by applying it to 55 published SI studies for water–oil and water–air experiments for a wide range of viscosity ratios, initial water content,

Table 1
Previously derived analytical solutions for two-phase flow with capillary effects. To resolve the influence of capillarity, all of them need to employ additional, non-essential assumptions that restrict their applicability. On the contrary, it can be shown (Schmid et al., 2011, Appendix A) that the solution given in McWhorter and Sunada (1990) is general. It can be viewed as the 'Buckley–Leverett Analogue' for counter-current SI (see Section 3.2). This makes the derivation of further specific solutions unnecessary.

Author and year	Assumption
Fokas and Yortsos (1982), Yortsos and Fokas (1983), Philip (1960), Chen (1988), Ruth and Arthur (2011), Wu and Pan (2003)	Specific functional forms for k_{rw}, k_{ro}, p_c
Kashchiev and Firoozabadi (2002)	Steady-state, i.e. $\frac{\partial S_w}{\partial t}=0$
Li et al. (2003)	Piston-like displacement, i.e. $F(x,t)=\frac{q_w(x^*,t)}{q_w(0,t)}$
Barenblatt et al. (1990), Zimmerman and Bodvarsson (1989), Tavassoli et al. (2005a, b), Mirzaei-Paiaman et al. (2011)	Approximate solution for the weak form
Handy (1960), Chen et al. (1995), Sanchez Bujanos et al. (1998), Rangel-German and Kovscek (2002)	Existence of an equivalent constant capillary diffusion coefficient
Ruth et al. (2007)	Self-similarity behaves according to specific functional form
Cil and Reis (1996), Reis and Cil (1993)	Linear capillary pressure, i.e. $\frac{dp_c}{dx}=\frac{p_c(S_0)}{L}$
Rasmussen and Civan (1998), Civan and Rasmussen (2001)	Asymptotic approximation of Laplace transformation for S_w
Zimmerman and Bodvarsson (1991)	Piecewise linear S_w profile