Journal of Petroleum Science and Engineering 96-97 (2012) 162–175

Contents lists available at SciVerse ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Pressure transient solutions to mixed boundary value problems for partially open wellbore geometries in porous media

Denis Biryukov, Fikri J. Kuchuk*

Services Techniques Schlumberger, 1 rue Henri Becquerel, 92142 Clamart Cedex, France

ARTICLE INFO

Article history:
Received 1 November 2011
Accepted 6 August 2012
Available online 30 August 2012

Keywords:
Mixed boundary value problems
Uniform-pressure boundary condition
Pressure transient testing
Partially penetrated wells
Limited entry wells
Multiple production sections

ABSTRACT

In this paper we present semi analytical pressure transient solutions for a limited-thickness open cylindrical production ring (often called limited entry or partially penetrated) on a non-permeable cylindrical wellbore in a porous medium which is used for fluid withdrawal (production) from the formation. Solutions are not readily available for the uniform-pressure boundary condition on the open cylindrical ring surface because such a condition creates mixed boundary value problem (MBVP) which is difficult to solve. It is well known that the approaches presented over the years for partially penetrated wells are based on the uniform-flux line source solutions for which the middle point, the equivalent-pressure point, or pressure averaging is used to approximate the uniform-wellbore-pressure boundary condition. Here we present exact pressure transient solutions obtained under assumption that the pressure on the open surface is uniform but *a priori* unknown and the rest of the wellbore surface is non-permeable (no-flow condition). Furthermore, we generalize our solution for the case of multiple production sections on the wellbore for both well testing and Wireline formation tester packer modules.

To solve MBVP, we write it in the form of a dual-integral equation. Next, using the special set of basis functions for representing the flux density on the open surface, we reduce it to an infinite system of linear algebraic equations with a symmetric diagonally dominated matrix. These properties of the matrix together with the singular nature of basis functions ensure very fast convergence of the truncated solutions to their limit, thus only a few equations can be considered and the system can easily be solved numerically.

The solutions presented in this paper are also compared with the middle point, equivalent-pressure point, and pressure-averaging solutions given in the petroleum literature.

1. Introduction

In this paper we present analytical pressure transient solutions for a limited-thickness open cylindrical interval(s) on a non-permeable cylindrical wellbore for production in a porous medium. This production/injection well geometry is often called limited entry or partially penetrated. The limited entry or partially penetrated terminology is actually not accurate because it implies that the well is not drilled through the whole oil-bearing formation. Most wells are normally drilled through the whole oil-bearing formation and most commonly a little bit further in the bottom of the formation to create a rat hole for heavy fluids and debris to be collected below the producing intervals. Many partially penetrated well geometries are due to partial perforations. Although it is not the best terminology, but we still continue using it because the terminology has been used in the petroleum literature since Muskat (1937). Therefore, limited entry or partially penetrated should be understood as opening (perforating) of selected production intervals over the sandface (casing) in fully formation-penetrating or partially formation-penetrating deviated, horizontal, and vertical wells to avoid or minimize

(1) commingled production,
(2) gas and water coning,
(3) conformance controlling,
(4) water encroachment over selected intervals, and
(5) perforation cost.

Furthermore, most gravel packed, screened, and slotted-liner completed wells are partially penetrated. Wells with multi-zone intelligent completion and inflow control devices (ICD) are partially penetrated. All horizontal and multilateral wells with or without multi-zone intelligent completions, inflow control

* Corresponding author. Tel.: +33 1 4537 7089; fax: +33 1 4537 2727.
E-mail address: kuchuk1@slb.com (F.J. Kuchuk).

0920-4105/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.petrol.2012.08.001

Nomenclature

C	wellbore storage constant (RB/psi, m^3/Pa)
c	compressibility (psi^{-1}, Pa^{-1})
erf	error function
h	formation thickness (ft, m)
g	unit impulse response
I	modified Bessel function of the first kind
J	Bessel function of the first kind
K	modified Bessel function of the second kind
k	permeability (md, m^2)
l	half-length (ft, m)
p	pressure (psi, Pa)
q	flow rate (STB/D, m^3/s)
r	radius or radial coordinate
s	Laplace transform variable
T	Chebyshev polynomial
t	time (h, s)
x	coordinate
y	coordinate
z	vertical coordinate
α	constant
β	constant
γ	constant
δ	constant
η	diffusivity for pressure
μ	viscosity (cp, Pa s)
ϕ	porosity (fraction)
τ	dummy variable
ξ	dummy variable
ω	Fourier frequency variable

Subscripts

a	pressure averaged
D	dimensionless
f	formation
h	horizontal
m	measured
o	initial or original
sf	sandface
u	uniform pressure
v	vertical
w	wellbore

Superscripts

-	Laplace transform
=	Laplace and Fourier transform

devices, gravel packings, screens, and/or slotted-liners are partially penetrated. Most openhole wells, which may be not drilled through the whole oil-bearing formation to minimize gas and/or water coning, create limited entry flow geometries. This applies for both horizontal and vertical wells. Wireline formation tester packer module interval tests also create partially penetrated flow geometries in both horizontal and vertical wells. Finally, it should be said that most of oil and gas production and injection wells are cased, cemented, and then perforated with shot densities much less than 100%. Therefore, any perforated well is partially penetrated. Moreover, shot densities in horizontal wells are much lower than those for the vertical wells. Since most horizontal wells are partially formation penetrating, distributed perforations in such wells further create multiple partially penetrated open sections for production/injection.

As discussed above, the partially open well geometry (partial penetration or limited flow entry) is one of the most common features of oil and gas wells and affects the steady-state and transient pressure behavior of wells considerably when the penetration ratio (the thickness of the open interval to the formation thickness) is less than 60%. As shown in Figs. 1 and 2, a partially open well geometry creates a mixed boundary-value problem, which is difficult to solve by exact analytical methods. Weber (1873) was the first to present a method for solving an electrical potential problem on a circular disk in a 3D medium, whereby the first order boundary corrections for such mixed boundary value problems may be obtained, provided the zeroth order solution is available. The technique assumes that boundaries are sufficiently far away compared to the thickness of the open interval, so that the flux distribution on the open interval does not change significantly (Ramakrishnan et al., 1995).

In the ground water hydrology and petroleum engineering literature, the pressure behavior of partially penetrated wells has been extensively investigated since the steady-state solution given by Muskat (1937). The first two pressure transient solutions for partially open wells were presented by Hantush (1957) and Nisle (1958) after the Muskat (1937) steady-state solution. Nisle (1958) used the line source approximation to derive the finite-length uniform-flux line source solution. The wellbore pressure given in these two papers was approximated at the middle point of the open section of the well. Brons and Marting (1961) in their well-known solution used the same method. Mixed boundary value problems (MBVP) have also been studied significantly in applied mathematics (see Snedon, 1966; Stakgold, 1972, 1979).

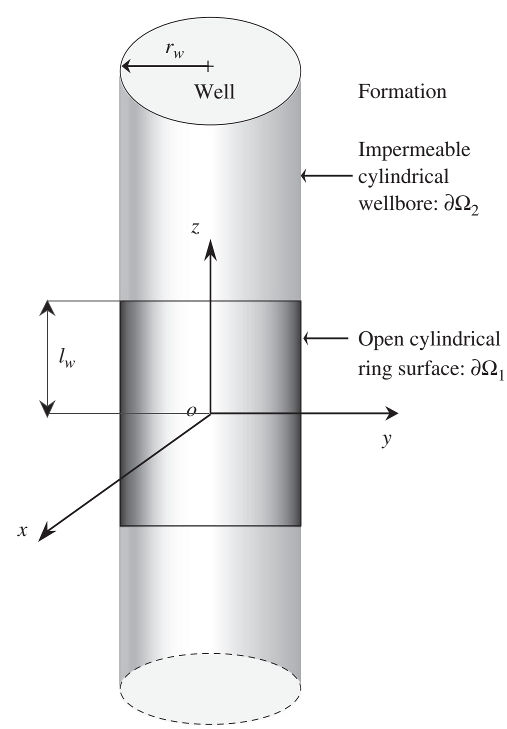

Fig. 1. Schematic representation of an open ring flow section on an impermeable cylindrical wellbore in a homogeneous porous medium (formation).