Journal of Petroleum Science and Engineering 86–87 (2012) 144–152

Contents lists available at SciVerse ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Interval inversion of well-logging data for automatic determination of formation boundaries by using a float-encoded genetic algorithm

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A R T I C L E I N F O

Article history:
Received 1 June 2011
Accepted 25 March 2012
Available online 2 April 2012

Keywords:
local inversion
interval inversion
global optimization
genetic algorithm
formation thickness

A B S T R A C T

In the paper a real-valued genetic algorithm is presented for solving the non-linear well-logging inverse problem. The conventional way followed in the interpretation of well-logging data is the formulation of the inverse problem in each measuring point separately. Since barely less number of unknowns than data are estimated to one point, a set of marginally overdetermined inverse problems have to be solved, which sets a limit to the accuracy of estimation. Describing the petrophysical (reservoir) parameters in the form of series expansion, we extend the validity of probe response functions used in local forward modeling to a greater depth interval (hydrocarbon zone) and formulate the so-called interval inversion method, which inverts all data of the measured interval jointly. Assuming an interval-wise homogeneous petrophysical parameter distribution, significantly smaller number of unknowns than data have to be determined. The highly overdetermined inverse problem results in accurate and reliable estimation of petrophysical parameters given for the whole interval instead of separate measuring points. For measuring the storage capacity of the reservoir, the formation thickness is also required to be estimated. As a new feature in well logging inversion methodology, the boundary coordinates of formations are treated as new inversion unknowns and determined by the interval inversion method automatically. Instead of using traditional linear inversion techniques, global optimization is used to avoid problems of linearization related to the determination of formation thicknesses. In the paper, synthetic and field examples are shown to demonstrate the feasibility of the interval inversion method.

1. Introduction

The information extracted from well-logging data is of great importance in reservoir engineering. The principal objectives of well log analysis are the identification of porous and permeable zones containing hydrocarbons and the determination of petrophysical parameters such as porosity, permeability, water saturation, shale content and specific volumes of mineral constituents. Besides the above quantities other important structural properties can also be determined by well-logging data processing. The thickness of the hydrocarbon-bearing formation plays also an important role in mapping the reservoir geometry and calculating of hydrocarbon reserves.

The petrophysical interpretation of well-logging data is traditionally solved by deterministic procedures that substitute data to explicit equations in order to determine petrophysical parameters separately (Serra, 1984; Asquith and Krygowski, 2004). The determination of the position of boundary between shale and permeable non-shale formations is done by manual operation. The most common well logs used for this purpose are natural gamma ray intensity and spontaneous potential logs supported by micro-resistivity logs for detailed study. The positions of formation boundaries are located by studying the shapes of well logs, which are influenced by many factors such as formation thickness, R_t/R_m ratio (R_t and R_m denote true and mud resistivities, respectively), type of probe, logging speed and other physical conditions, *e.g.* the statistical variation of gamma ray counts (Lynch, 1962).

The most advanced way for extracting petrophysical parameters from well-logging data nowadays is the use of geophysical inversion methods, which process data acquired in a certain measuring point so as to determine petrophysical model parameters only to that point. This local inversion technique represents a narrow type of overdetermined inverse problem, because the number of data measured by different probes is slightly more than that of the unknowns. This leads to a set of separate inversion runs in adjacent measuring points for the logging interval. Many well log interpretation systems are based on this inversion methodology, *e.g.* Schlumberger Global (Mayer and Sibbit, 1980), Gearhart Ultra (Alberty and Hashmy, 1984) and Baker Hughes Optima (Ball et al., 1987). Along with several advantages, such as quickness and good vertical resolution, the method has some limitations as well. The marginal overdetermination of the local inverse problem sets a limit to the accuracy and reliability of the estimation.

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0920-4105/$ – see front matter
doi:10.1016/j.petrol.2012.03.028

On the other hand, the inversion method does not support the determination of formation thicknesses, because they are not included explicitly in the response functions attached to the local forward problem. Log analysts are still restricted to handle this problem manually in a distinct (non-automatic) procedure. However, the complete data set collected in a longer depth-interval does contain information also on the formation boundaries that can be extracted by using an appropriately defined inversion method.

The proposed well-logging inversion methodology was developed by Dobróka and Szabó (2002). Describing the petrophysical parameters in the form of series expansion, the validity of response functions used in local forward modeling can be extended to a greater depth interval. The so-called interval inversion procedure is based on the use of the depth dependent response functions, in which well-logging data of an optional depth interval are inverted jointly in order to give an estimate for petrophysical parameters to the same interval. The joint inversion procedure can be formulated to have order(s) of magnitude greater over-determination (data-to-unknown) ratio compared to local inversion, which results in a significant improvement in the quality of interpretation (Dobróka and Szabó, 2002, 2005; Dobróka et al., 2007, 2008; Szabó, 2004).

An advantageous property of the interval inversion technique is its capability to treat increasing number of inversion unknowns without significant decrease of overdetermination ratio. As a new feature in well logging inversion methodology, we specified the boundary coordinates of formations as inversion unknowns, which can be determined simultaneously with the conventional petrophysical parameters. This inversion strategy allows a more objective determination of formation thicknesses affecting greatly the accuracy of reserve calculation.

2. Derivation of petrophysical parameters from well-logging data

Well-logging data sets consist of several lithology, porosity and saturation sensitive measurements. A typical combination of well logs used in hydrocarbon exploration is presented in Table 1. In this section, we overview the forward modeling of well-logging data and the principles of the local inverse problem based on different optimization techniques.

2.1. The forward problem

The mathematical relationship between the petrophysical model and well-logging data is called response function. The following linear response functions were used for computing wellbore data listed in Table 1 including correction of hydrocarbon and shale effect

$$GR = \Phi\left[GR_{mf}S_{x0} + GR_{hc}(1-S_{x0})\right] + V_{sh}GR_{sh} + V_{sd}GR_{sd}, \quad (1)$$

$$SP = \Phi(1-S_{x0})SP_{hc} + V_{sh}(SP_{sd}-SP_{sh}) + SP_{sd}, \quad (2)$$

$$\Phi_N = \Phi\left[\Phi_{N,mf}S_{x0} + K_1(1-S_{x0})\right] + V_{sh}\Phi_{N,sh} + V_{sd}\Phi_{N,sd}, \quad (3)$$

$$\rho_b = \Phi\left[\rho_{b,mf}S_{x0} + K_2(1-S_{x0})\right] + V_{sh}\rho_{sh} + V_{sd}\rho_{sd}, \quad (4)$$

$$\Delta t = \Phi\left[\Delta t_{mf}S_{x0} + (1-S_{x0})\Delta t_{hc}\right] + V_{sh}\Delta t_{sh} + V_{sd}\Delta t_{sd}, \quad (5)$$

where Φ denotes porosity, S_{x0} and S_w are water saturation of the invaded and undisturbed zones respectively, V_{sh} is shale volume and V_{sd} is the volume of sand. The rest of the parameters appearing in Eqs. (1)–(5) are treated as constant representing physical properties of mud filtrate (*mf*), hydrocarbon (*hc*), shale (*sh*) and sand (*sd*). K_1 and K_2 are hydrocarbon type constants. For computing resistivity data, the non-linear Indonesian formulae were applied (Poupon and Leveaux, 1971)

$$\frac{1}{\sqrt{R_d}} = \left[\frac{V_{sh}^{(1-0.5V_{sh})}}{\sqrt{R_{sh}}} + \frac{\left(\sqrt{\Phi}\right)^m}{\sqrt{aR_w}}\right]\left(\sqrt{S_w}\right)^n, \quad (6)$$

$$\frac{1}{\sqrt{R_s}} = \left[\frac{V_{sh}^{(1-0.5V_{sh})}}{\sqrt{R_{sh}}} + \frac{\left(\sqrt{\Phi}\right)^m}{\sqrt{aR_{mf}}}\right]\left(\sqrt{S_{x0}}\right)^n, \quad (7)$$

where m denotes the cementation exponent, n is the saturation exponent and a is the tortuosity factor representing textural properties of rocks (Tiab and Donaldson, 2004). The textural constants can be estimated from literature or determined by using the interval inversion method (Dobróka and Szabó, 2011). It is clearly seen that the above response equations do not contain the boundary coordinates of formations and they are only dependent on petrophysical properties of the formation in the near vicinity of the given measuring point. The information inherent in data observed in one measuring point is not sufficient to extract the formation thicknesses by any local inversion methods.

Table 1
Well log types used in hydrocarbon exploration and their specification.

Code	Symbol	Name of well log	Sensitive to	Unit
SP	SP	Spontaneous potential		mV
GR	GR	Natural gamma ray intensity		API
K	K	Potassium (spectral gamma ray intensity)		Percent
U	U	Uranium (spectral gamma ray intensity)	Lithology	ppm
TH	TH	Thorium (spectral gamma ray intensity)		ppm
PE	P_e	Photoelectric absorption index		Barn/electron
CAL	d	Caliper		Inch
CN	Φ_N	Compensated neutron porosity		Porosity unit
DEN	ρ_b	Density	Porosity	g/cm^3
AT	Δt	Acoustic travel time		μs/m
RMLL	R_{mll}	Microlaterolog	Saturation	ohmm
RS	R_s	Shallow resistivity		ohmm
RD	R_d	Deep resistivity		ohmm

2.2. The local inverse problem

In formulating the inverse problem, we introduce the column vector of the local model parameters for shaly–sand formations as

$$m = [\Phi, S_{x0}, S_w, V_{sh}, V_{sd}]^T, \quad (8)$$

where T is the symbol of transpose. Well-logging data measured at the same measuring point are also represented in a column vector

$$d^{(o)} = [SP, GR, \Phi_N, \rho_b, \Delta t, R_s, R_d]^T. \quad (9)$$

If the size of vector $\boldsymbol{d}^{(o)}$ is larger than that of vector $\boldsymbol{m}$, the inverse problem is called overdetermined. The calculated data are connected to the model nonlinearly as

$$d^{(c)} = g(m), \quad (10)$$

where $\boldsymbol{g}$ represents the set of response functions, which is used to predict well-logging data locally in the measuring point (*e.g.* Eqs. (1)–(7) represent an empirical connection between data and model). Since the number of data is slightly more than that of the model parameters, this formulation leads to a marginally