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Modified Kozeny–Carmen correlation for enhanced hydraulic flow unit characterization

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ABSTRACT

Although several techniques have been proposed to predict permeability using porosity–permeability relationships, the Kozeny–Carmen (K–C) correlation is the most widely acceptable methodology in the oil industry. Amaefule et al. (1993) modified that correlation introducing the concept of Reservoir Quality Index (RQI) and Flow Zone Indicator (FZI) to enhance its capability to capture the various reservoir flow behavior based on its respective characters. Yet, there are challenges in using the original correlation due to its inherent limitations and over simplified assumptions that prevent accurate Hydraulic Flow Unit (HFU) definitions. This research addresses some of those shortcomings and proposes a modified K–C correlation by handling the tortuosity term in a more robust manner. Core data from major carbonate reservoirs in Saudi Arabia is used to test the model. Additional data sets obtained from literature on sandstone reservoirs are used as well to demonstrate the global applicability of the proposed model. Results show that more permeability variations are to be expected within a given HFU. Moreover, the conventional model underestimates permeability values within a specific HFU significantly in comparison with the new model.

1. Introduction

Permeability is one of the most important parameters to quantify in any reservoir rock. Its importance arises due to the major role it plays during the development phase of any reservoir. For many years, various techniques have been proposed to measure permeability. Literature shows that permeability can be measured by three major techniques; (1) well testing, (2) routine core analysis, and (3) formation testers (Ahmed et al., 1991). Ahmed et al. (1991) provided a critical and detailed review of permeability measurement techniques and their interrelationships.

During any reservoir simulation study, permeability perdition is a very critical and perhaps the most challenging task. In the early stage of the industry, simple permeability–porosity transformations were generated to estimate permeability at un-cored wells. However, such simple relationships were unreliable and results were not in good agreement with field data. Hence, many models have been proposed to predict permeability by incorporating many parameters other than effective porosity.

Nelson (1994) made an extensive review of most permeability models available two decades ago. Haro (2004) also made a detailed comparison of four permeability models (Windland, Kozeny–Carmen, Civan and Lucia). He concluded that the K–C model is the most practical correlation that has good theoretical bases. However, The K–C correlation has inherent limitations since it was derived based on the assumption that porous media can be represented as a bundle of unconnected capillary tubes having identical radius and constant cross-sectional area (Civan, 2002).

In 1993, Amaefule et al. (1993) introduced for the first time the concept of reservoir quality index (*RQI*) and flow zone indicator (*FZI*) to identify HFU based on the K–C model. In this regard, Amaefule's technique is recognized as a very simple, practical, and widely used established technique (Amaefule et al., 1993; Davies and Vessell, 1996; Shenawi et al., 2007). However, the developed technique suffers from the same limitations of the original K–C model that prevent accurate HFU identification.

In this article, a modification of the K–C correlation is proposed by handling the tortuosity term in a more representative manner. Core data from major carbonate reservoirs in Saudi Arabia is used to test the model. Additional data sets obtained from literature are used as well to show the global applicability of the proposed model.

2. Kozeny–Carmen (K–C) correlation

Many models have been proposed to estimate permeability from effective porosity and other relevant parameters. One of the earliest is the Kozeny model (Kozeny, 1927). Its correlation expresses the permeability as a function of effective porosity, tortuosity and specific surface area. It was able to derive correlation by considering the

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porous medium as a bundle of tortuous capillary tubes with the same radius. By combining Poiseuille's equation with Darcy's law and solving for permeability (k), Kozeny obtained the following relationship:

$$k = \frac{\phi}{8\tau} r^2 \tag{1}$$

where (k) is permeability in μm^2, (τ) is the tortuosity, (ϕ) is the effective porosity in fraction and (r) is the radius of the capillary tubes in μm.

The equation was later modified by Carmen (Carman, 1937) and the popular form is given by the following formula:

$$k = \left(\frac{1}{f_g \tau S_{V_{gr}}^2}\right) \frac{\phi^3}{(1-\phi)^2} \tag{2}$$

where (k) is permeability in μm^2, (f_g) is the shape factor, (τ) is the tortuosity, ($S_{V_{gr}}$) is the specific surface area of the grain in μm^{-1} and (ϕ) is the effective porosity in fraction.

The K–C correlation was developed based on the concept of average pore throat size. It was found that this correlation works best for synthetic porous media where pore systems are homogenous and easy to quantify. However, this equation does not work properly in heterogeneous and complex pore systems (Ahmed et al., 1991; Babadagli and Al-Salmi, 2004; Francisco et al., 2009).

3. Conventional HFU characterization technique

Amaefule et al. (1993) developed a technique to characterize HFU using the K–C model based on the concept of mean hydraulic radius and flow units. Tiab (2000) defined a hydraulic flow unit as "a continuous body over a specific reservoir volume that practically possesses consistent petrophysical and fluid properties, which uniquely characterize its static and dynamic communication with the wellbore".

Theoretically, RQI versus the ratio of pore volume to grain volume (ϕ_z) plotting should yield a straight line on log–log plot with a unit slope line. Rock samples with similar FZI values will be positioned on a unit slope line forming a HFU. Other rock samples with different FZI values will lie on other parallel lines. Unfortunately, this is not always the case. In fact, Civan (2002) and Haro (2004) showed that natural rock systems tend to show various slopes rather than having a fixed slope as suggested by Amaefule et al. (1993) and the K–C-model.

4. Proposed modification to the K–C model

A modified K–C model is developed by handling the tortuosity term in more robust approach. The tortuosity can be approximated accurately from electrical property measurements and effective porosity. The following sections illustrate the theoretical development of the proposed model and its application in characterizing HFU.

4.1. Tortuosity role in the K–C model

Tortuosity (τ) is defined as the squared ratio of the path traveled by a fluid particle through a porous medium (L_a) to the actual length of the porous medium, L (Rose and Bruce, 1949; Wyllie and Rose, 1950) which can be expressed mathematically as:

$$\tau = \left(\frac{L_a}{L}\right)^2 \tag{3}$$

A relationship between tortuosity, formation resistivity factor (F_R) and cementation exponent (m) has been derived using theoretical approaches (Wyllie and Rose, 1950; Winsauer et al., 1952). Wyllie and Rose (1950) were able to develop the following relationship:

$$\tau = (F_R * \phi)^2 \tag{4}$$

Since (F_R) can be approximated using Archie's equation (Archie, 1942) as:

$$F_R = \frac{a}{\phi^m} \tag{5}$$

where, (a) is the lithology factor and (m) is the cementation exponent. Eq. (4) then can be written as:

$$\tau = \left(\frac{a}{\phi^{m-1}}\right)^2 \tag{6}$$

Eq. (6) demonstrates the nonlinear relationship between tortuosity and porosity. Theoretically, a bundle of capillary tubes would have (a) and (m) equal to one. In that case, tortuosity would also be one. Similarly, as porosity approaches a hypothetical value of 100% (the common range of porosity in petroleum reservoirs is between 10% and 20%, Tiab and Donaldson, 2004), tortuosity would also approach one, as expected. Fig. 1 shows the tortuosity variation with porosity for different m values where the above mentioned phenomena can be explained. Moreover, if we increase the m value, the tortuosity–porosity relationship becomes more nonlinear.

4.2. Verification of tortuosity model using experimental data

Hagiwara (1986) data set was used to validate the approximation of tortuosity using Eq. (6). Hagiwara proposed a model to estimate permeability using a theoretical approach which is given by the following relation:

$$k = c\,\phi^m \langle R^2 \rangle \tag{7}$$

where, ($\langle R^2 \rangle$) is the average pore throat radius squared in μm^2 and (c) is a constant. Eq. (7) can be rewritten as:

$$k = c\frac{\phi}{(1/\phi^{m-1})} \langle R^2 \rangle = c\frac{\phi}{\tau_H} \langle R^2 \rangle \tag{8}$$

One can notice that Eq. (8) is similar to Kozeny's equation (Eq. (1)) but with different tortuosity definition (i.e. c is equivalent to 1/8 and τ is equivalent to $(\tau_H)^2$). Consequently, the tortuosity model in Eq. (6) (assuming a = 1 for simplicity) is employed in

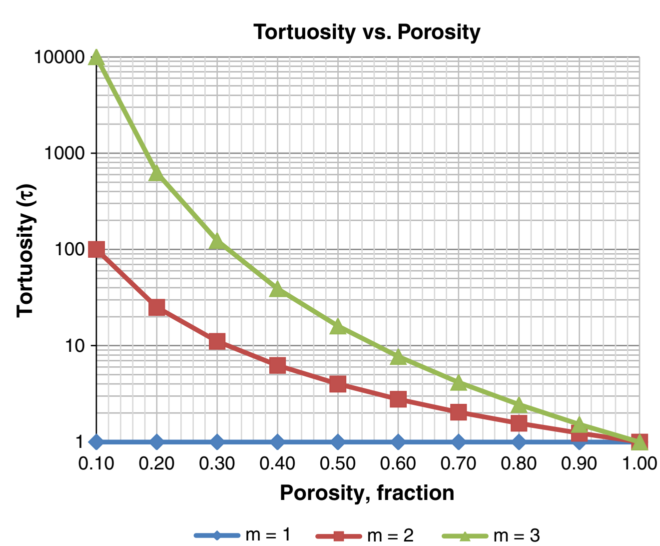

Fig. 1. Tortuosity variation with porosity at various cementation exponent values.