Journal of Petroleum Science and Engineering 76 (2011) 93–99

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Viscoelastic flow and species transfer in a Darcian high-permeability channel

O. Anwar Bég [a,*], O.D. Makinde [b]

[a] Biomechanics, Biotechnology and Magnetohydrodynamics, Mechanical Engineering, Department of Engineering and Mathematics, Sheffield Hallam University, Sheffield, S1 1WB, England, UK

[b] Institute for Advance Research in Mathematical Modelling and Computations, Cape Peninsula University of Technology, P. O. Box 1906, Bellville 7535, South Africa

ARTICLE INFO

Article history:
Received 13 May 2010
Accepted 10 January 2011
Available online 21 January 2011

Keywords:
viscoelastic flow
mass transfer
upper convected Maxwell (UCM) fluid
Deborah number
MAPLE
petroleum flows

ABSTRACT

We study the two-dimensional steady, laminar flow of an incompressible, viscoelastic fluid with species diffusion in a parallel plate channel with porous walls containing a homogenous, isotropic porous medium with high permeability. The Darcy model is employed to simulate bulk drag effects on the flow due to the porous matrix. The upper convected Maxwell model is implemented due to its accuracy in simulating highly elastic fluid flows at high Deborah numbers. The conservation equations are transformed into a pair of couple nonlinear ordinary differential equations which are solved numerically using efficient 6th order Runge–Kutta shooting quadrature in the computer algebra package system MAPLE. The effects of Darcy number (Da), Deborah number (De), Schmidt number (Sc) and transpiration Reynolds number (Re_T) on velocity and species concentration distributions and also wall shear stress and concentration gradients are examined in detail. The study finds applications in petroleum filtration dynamics, hydrocarbon fluid flow in geosystems, oil spill contamination in soils and also chemical engineering technologies.

1. Introduction

Viscoelastic flows are encountered in numerous areas of petrochemical, biomedical and environmental engineering including polypropylene coalescence sintering (Scribben et al., 2006), dynamically-loaded journal bearings (Tichy, 1996), blood flow (Chien et al., 1975; Vlastos, 1998) and geological flows (Wouter et al., 2005). A wide range of mathematical models have been developed to simulate the nonlinear stress–strain characteristics of such fluids which exhibit both viscous and elastic properties. A detailed discussion of such models which include the upper convected Maxwell model, the Walters-B model and the Reiner–Rivlin second-order model is provided in Zahorski (1982). For highly elastic fluids such as polymer melts, the upper convected Maxwell (UCM) model has proved to be very reliable. This viscoelastic flow model is a generalization of the Maxwell material model for the case of *large* deformations and was derived by Oldroyd using an upper convected time derivative (Oldroyd, 1950). Many theoretical, numerical and experimental studies have utilized this model. Horikawa (1987) presented finite difference solutions using a perturbation method for the flow of a UCM fluid around an inclined circular cylinder of finite length showing that the viscoelastic tends to flow axially in the vicinity of the cylinder. Chiba et al. (1988) investigated analytically the effects of a wall transpiration of the UCM fluid flow via a porous-walled tube. Larson (1988) used a similarity transformation to model the UCM flow in an infinitely long cylinder whose surface has a velocity that increases in magnitude linearly with an axial coordinate showing that with an increasing elasticity of the fluid normal stress gradients in an elastic boundary layer near the accelerated surface aid in offseting inertially-generated negative axial pressure gradients. Roberts and Walters (1992) obtained spectral numerical solutions for the three-dimensional flow of a UCM fluid in a journal bearing, operating under *static* loading conditions, showing that a relaxation time of the order of 10^{-4} s is needed prior to viscoelasticity enhancing the load-bearing capacity. Maders et al. (1992) used a decoupled finite element method to study the flow of a polymeric UCM fluid in a 2-dimensional convergent geometry. Brown et al. (1993) have studied the linear stability of the planar Couette flow of a UCM fluid using a mixed finite-element method obtaining stabile calculations for the values of the *Deborah number* in excess of 50. Khayat (1994) used a perturbation technique to analyze the two-dimensional incompressible viscoelastic UCM flow between two parallel plates, with two straight free boundaries. The hyperbolic partial differential equations were solved using an implicit finite-difference procedure. Rahaman (1997) investigated the transient UCM viscoelastic flow in a rectangular duct. Avgousti and Renardy (1998) studied computationally the hydrodynamic stability of the eccentric Dean flow of a UCM fluid. Xue et al. (1998) employed a 3-dimensional finite volume numerical solver to model Lagrangian transient extensional flow of both a Phan-Thien Tanner (PTT) viscoelastic and a UCM viscoelastic fluid in a rectangular duct with a sudden contraction is carried out using a three-dimensional (3-D) finite volume method (FVM). Further studies of the viscoelastic UCM flows were described by

* Corresponding author.
E-mail addresses: o.beg@shu.ac.uk (O.A. Bég), makinded@cput.ac.za (O.D. Makinde).

0920-4105/$ – see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.petrol.2011.01.008

Notation

C	concentration of species diffusing in fluid
C_w	concentration at channel center ($y=0$).
C_H	concentration at both plates (i.e. at $y=H/2$, $y=-H/2$)
D	species diffusivity
H	channel width
K	permeability of the porous medium
u	velocity in x-direction
v	velocity in y-direction
$V/2$	suction velocity at the plates
x	coordinate along channel center-line (parallel to plates)
y	coordinate normal to channel center-line
λ	relaxation time of the UCM fluid
ν	kinematic viscosity of the UCM fluid

Dimensionless parameters

Da	Darcy number
De	Deborah number
F	dimensionless stream function
Re_T	transpiration (suction) Reynolds number (>0)
Sc	Schmidt number
X	dimensionless coordinate along channel center-line (parallel to plates)
Y	dimensionless coordinate normal to channel center-line
ϕ	dimensionless concentration function

Van Os and Gerritsma (2001); Ghosh and Sengupta (2002) with magnetohydrodynamic effects and from a continuous stretching surface by Sadeghy et al. (2005). Evans (2005) discussed the steady planar flow of the UCM fluid for the re-entrant corners with obtuse angles, obtaining a class of similarity solutions associated with the inviscid flow equations which arise from the dominance of the upper convective stress derivative in the constitutive equations. He also identified two classes of the boundary-layer structure, namely the Renardy single-layer structure and a new double-layer boundary layer structure. Sadeghy et al. (Sadeghy, 2006) used a Chebyshev pseudo-spectral collocation-point method to simulate the two-dimensional stagnation-point flow of the UCM viscoelastic fluid indicating a thickening of the boundary layer and a drop in the wall skin friction coefficient with higher elasticity effects. Hayat et al. (2006) used the homotopy analysis method to obtain series solutions for the hydromagnetic boundary layer flow of a UCM fluid over a porous stretching sheet.

The above studies all omitted *any consideration of porous media* despite the frequent presence of such media in many petroleum and chemical engineering operations and systems. Filtration systems, petroleum geosystems, packed bed reactors and foodstuffs are several examples of porous media in which viscoelastic flows may occur. Generally the Darcy model is employed for low-velocity flows in porous media and relates the pressure drop in the porous medium to a linear drag force. Wissler (1971) gave the first analytical explanation for the elongation stresses developed in the viscoelastic flow in a Darcian porous medium, presenting a third-order perturbation analysis and showing that viscoelastic (e.g. polymer and hydrocarbon derivatives) experience a reduced mobility in porous media. James and Mclaren (1975) discussed experiments relating to the measurements of the pressure drop and flow rate for dilute viscoelastic solutions of polyethylene oxide flowing through beds of packed beads i.e. porous media, showing that viscoelasticity was most prevalent at moderate flow rates. A reduced viscoelastic effect at higher flow rates was attributed to the dominance of extensional stresses in this regime. Deiber and Schowalter (1981) performed experiments on the flow of dilute aqueous solutions of a polyacrylamide in a tube with sinusoidal axial variations in diameter as a porous medium flow model, showing that Lagrangian unsteadiness generates an increase in resistance to flow through the sinusoidal channel relative to that predicted for a purely viscous fluid. Two other excellent investigations of the viscoelastic flow in Darcian porous media include the articles by Durst et al. (1987) and the finite element study by Talwar and Khomami (1992). Tan and Masuoka (2005) used a modified Darcy's law for the Oldroyd-B viscoelastic fluid to study Stokes' first problem in a porous half space using a Fourier sine transform. They found that the boundary layer thickness has a limited value and deviates from the purely fluid case. Several studies have considered specifically the flow of Maxwell viscoelastic fluids in porous media. De Haro et al. (1996) used a volume averaging approach to study Maxwell flow in a rigid porous medium deriving the momentum equation with a time-dependent permeability tensor. They simulated the viscoelasticity effects by transforming the model to the frequency domain via a temporal Fourier transform and presented closed-form solutions for a porous medium modeled as a bundle of capillary tubes. Other studies include the papers by del Río et al. (1998) and Lopez et al. (2003). Numerous technological applications exist wherein both viscoelastic flow and mass (species) diffusion take place, including the separation systems, polymer processing, haemodynamics, petroleum displacement in reservoirs etc. Flows may be both laminar or turbulent. Cho and Hartnett (1981) employed the successive approximation technique to investigate the mass transfer entry length and maximum mass transfer reduction asymptote for the drag-reducing viscoelastic fluids obtaining a good correlation with the empirical mass transfer results for the predicted mass transfer rates and showing approximately 56–75% reduction in the mass transfer rate compared to the Newtonian values at the same Reynolds and Schmidt numbers. The transient species transfer in a viscoleastic tube flow was studied by Dalal and Mazumder (Dalal, 1998). Lo et al. (2003) studied the mass diffusion in curdlan viscoelastic gels. A detailed analysis has also been presented by Ramakrishnan (2004) of the non-Fickian species transfer in the polymeric viscoelastic flows. Herein we extend to consider the non-reactive version of the study of Hayat and Abbas (2007) but with the porous drag effects considered, to present extensive numerical solutions to the flow and dispersion of a species in a UCM viscoelastic-fluid saturated regime between the parallel plates and wall transpiration. The case of the low Deborah number (De) i.e. weak elasticity, is considered. The shooting iteration technique together with the Runge–Kutta sixth-order integration scheme is employed to solve the transformed system of the nonlinear ordinary differential equations. Such a study we believe constitutes an important addition to the literature on the rheological flows in the petroleum geosystem applications.

2. Hydromechanics of the UCM viscoelastic fluid

Zahorski (1982) has discussed extensively the mathematical aspects of the UCM viscoelastic model. This model is the most elementary of the nonlinear viscoelastic models which accounts for frame invariance in the nonlinear flow regime. It amounts to a succinct amalgamation of the Newtonian law for viscous fluids and the derivatives of the Hooke's law for elastic solids and cannot simulate more complex effects which are reproduced in the more elaborate viscoelastic formulations. Nevertheless in simple engineering flows, the UCM model is easily implemented and leads to relatively fewer stability and convergence problems in computation. The UCM model simulates purely elastic fluids with shear-